Hybrid Oscillator-Qubit Quantum Processors: Instruction Set Architecture, Abstract Machine Models, and Applications

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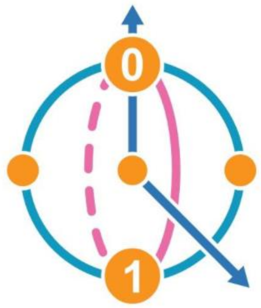
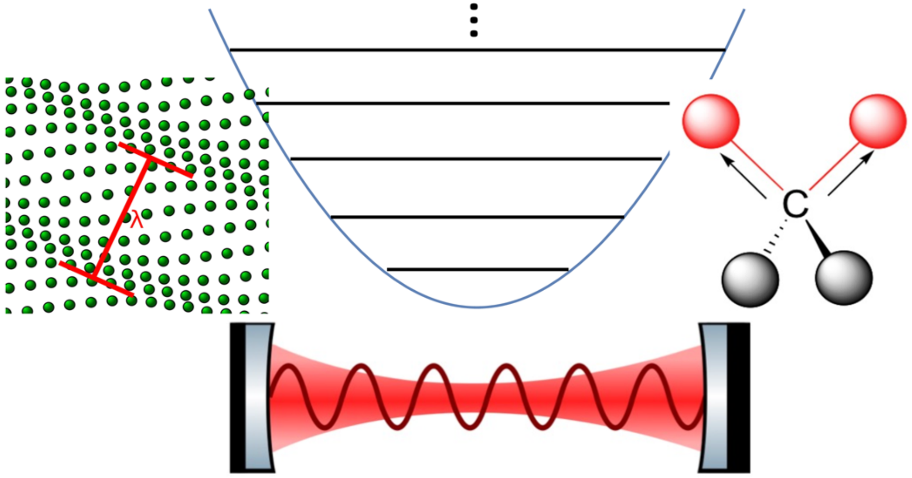
Qubit



$[X, Y] = 2iZ$

Bosonic modes (quantum oscillators)



- **Fock state, energy eigenstate**

$$\hat{n}|m\rangle = m|m\rangle; \qquad m = 0, 1, 2, 3, \ldots$$

- **Creation annihilation operator**

$$\begin{aligned} a^\dagger|m\rangle &= \sqrt{m+1}|m+1\rangle \\ a|m+1\rangle &= \sqrt{m+1}|m\rangle \\ [a, a^\dagger] &= 1 \\ \hat{n} &= a^\dagger a. \end{aligned}$$

- **Bosonic modes are ubiquitous:**
  - Vibrational modes of trapped ions or molecules
  - Phonons in solids
  - Quantum optics or fields

- **Coherent state** $|\alpha\rangle = e^{\alpha a^\dagger - \alpha^* a}|0\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha a^\dagger}|0\rangle$

$$\begin{aligned} \hat{x} &= \lambda_x(a + a^\dagger) \\ \hat{p} &= -i\lambda_p(a - a^\dagger) \end{aligned}$$

$[\hat{x}, \hat{p}] = i$

- **Infinite-dimensional (in principle)**

$$H_0 = \omega_{\mathrm{R}}\left[\hat{n} + \frac{1}{2}\right]$$

- The **physics** of oscillators are well known;
- Their **computational** power is much less well-known

Outline

- Why Need Quantum Computers Made with Oscillators + Qubits?
- Theoretical Challenges for Hybrid Quantum Processors
- What are AMM and ISA and Why are They Important?
- ISA and AMM for Hybrid Quantum Processors
- Algorithms and Applications
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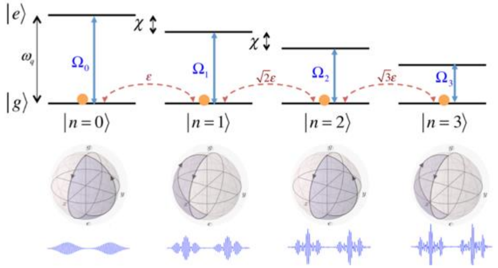
Why need quantum computers made with oscillators + qubits?

Theoretical needs

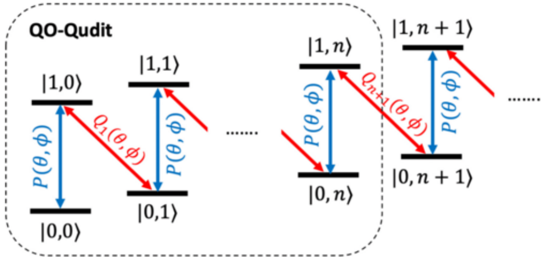
- Spin, boson, fermion
- Large dimensionality, resource efficient qudits.

SNAP gate universal control



Phys. Rev. A **92**, 040303(R) (2015)

Qubit-oscillator qudits

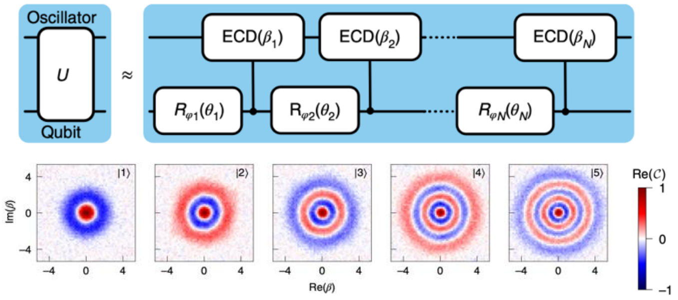


Phys. Rev. A 104, 032605 (2021).

Experimental advancement

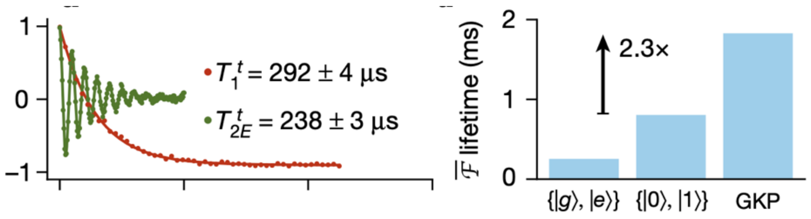
- Control: cQED cavity, trapped ions, boson sampling
- Bosonic codes

ECD Universal Control



Nature Physics **18**, 1464–1469 (2022).

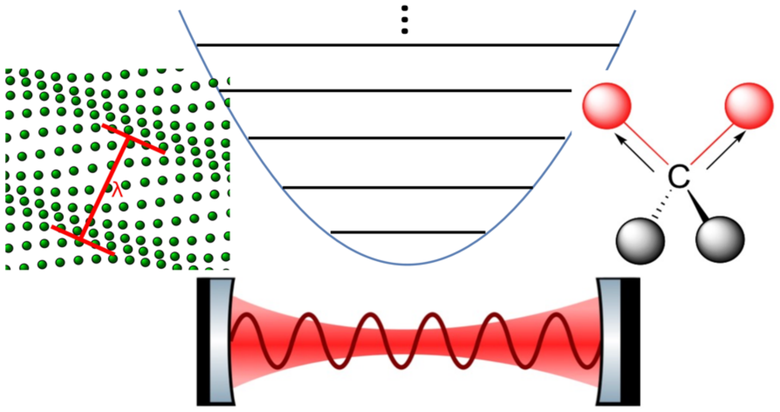
Bosonic Codes Break-even



Nature **616**, 50–55 (2023). Nature **616**, 56–60 (2023).

Application driven

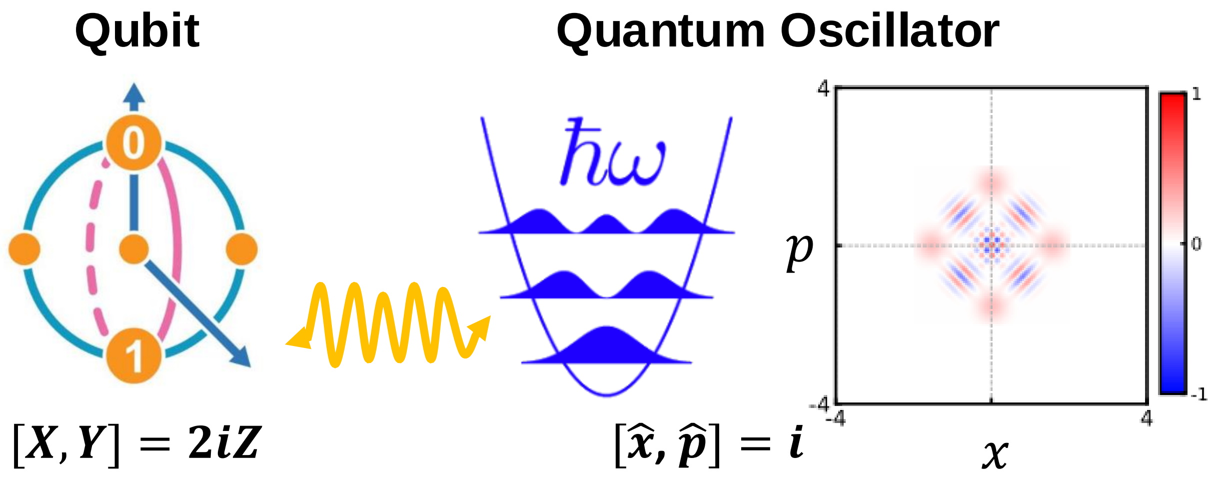
- Bosonic matter (vibrations, phonons, quantum fields,…) is difficult to simulate on qubit-based computers.
- Many problems are continuous in nature: optimization…



$$a^\dagger|m\rangle = \sqrt{m+1}|m+1\rangle$$

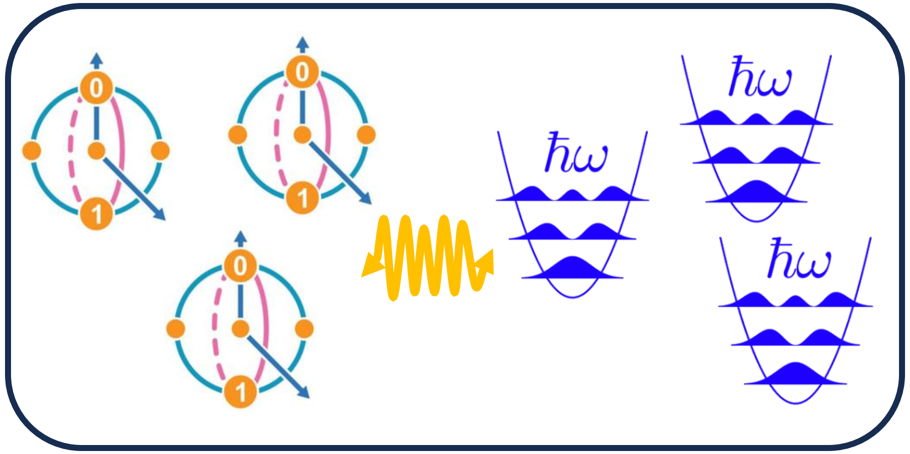
Theoretical challenges of oscillator-qubit quantum processors

- Universal control of oscillators
- Hybridize qubits and oscillators



- **Beyond physical layer:**
  - How to program?
  - How to reason about their computational power?
  - How to perform resource estimation?

A hybrid quantum computer



- Need an **abstract theoretical model for the hybrid QC**
- Need **universal instructions to program**

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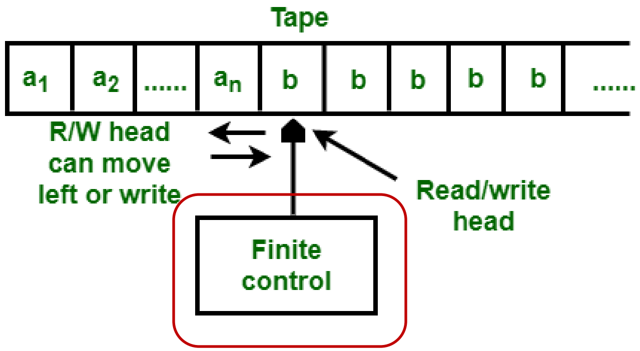
What are AMM and ISA?

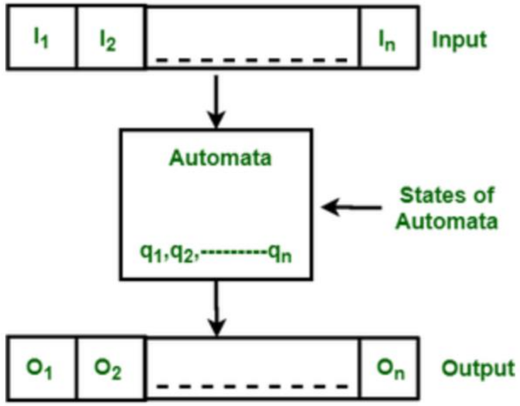
- AMM: Abstract Machine Model

AMM for classical computing machines

Turing machine



Finite automata



- ISA: Instruction Set Architecture

Instructions for EDSAC

A n — Add the number in storage location *n* into the accumulator.

E n — If the number in the accumulator is greater than or equal to zero execute next the order which stands in storage location *n*; otherwise proceed serially.

Z — Stop the machine and ring the warning bell.

[Hennessy and Patterson. Computer Architecture: A Quantitative Approach, 5th Edition.]

Wilkes and Renwick

Selection from the List of 18 Machine Instructions for the EDSAC (1949)

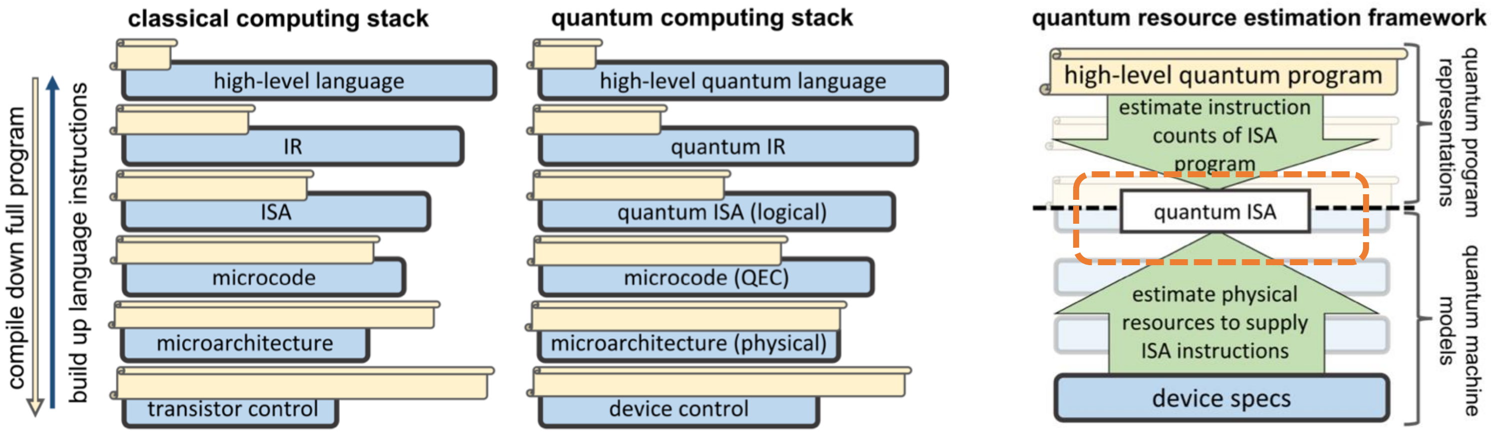
Instructions for 80x86

load
conditional branch
compare
store
add
and
sub
move register-register
call
return

How about for quantum computers?

AMM and ISA: Why are They Important?

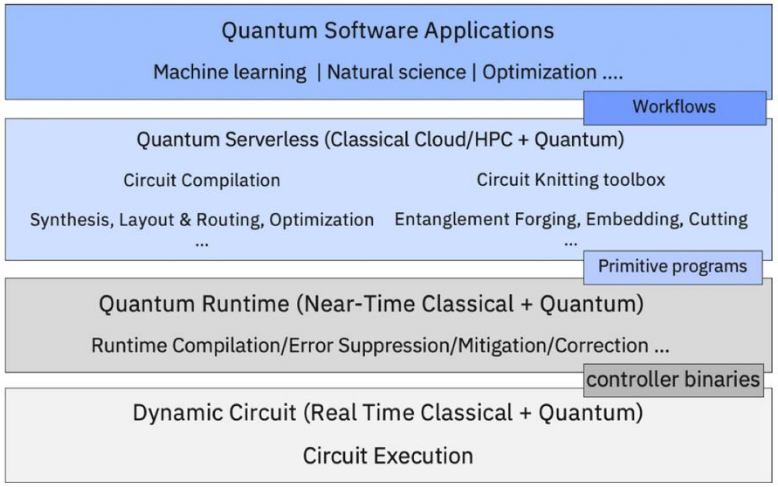
- Architecture stack for classical and qubit-based quantum computers



Beverland et al. Assessing requirements to scale to practical quantum advantage. (Microsoft, 2022)

- **Instruction Set Architecture (ISA)** bridges the high-level applications and low-level physical/device layers.

AMM and ISA for hybrid oscillator-qubit processors are unknown!



Bravyi et al. J. Appl. Phys. 132, 160902 (2022). IBM.

Quantum computation of stopping power for inertial fusion target design

[arXiv:2308.12352]

Nicholas C. Rubin,[1,*] Dominic W. Berry,[2,†] Alina Kononov,[3] Fionn D. Malone,[1] Tanuj Khattar,[1] Alec White,[4] Joonho Lee,[1,5] Hartmut Neven,[1] Ryan Babbush,[1,‡] and Andrew D. Baczewski[3,§]

nature communications

Lee *et* al. (2023)

Article

https://doi.org/10.1038/s41467-023-37587-6

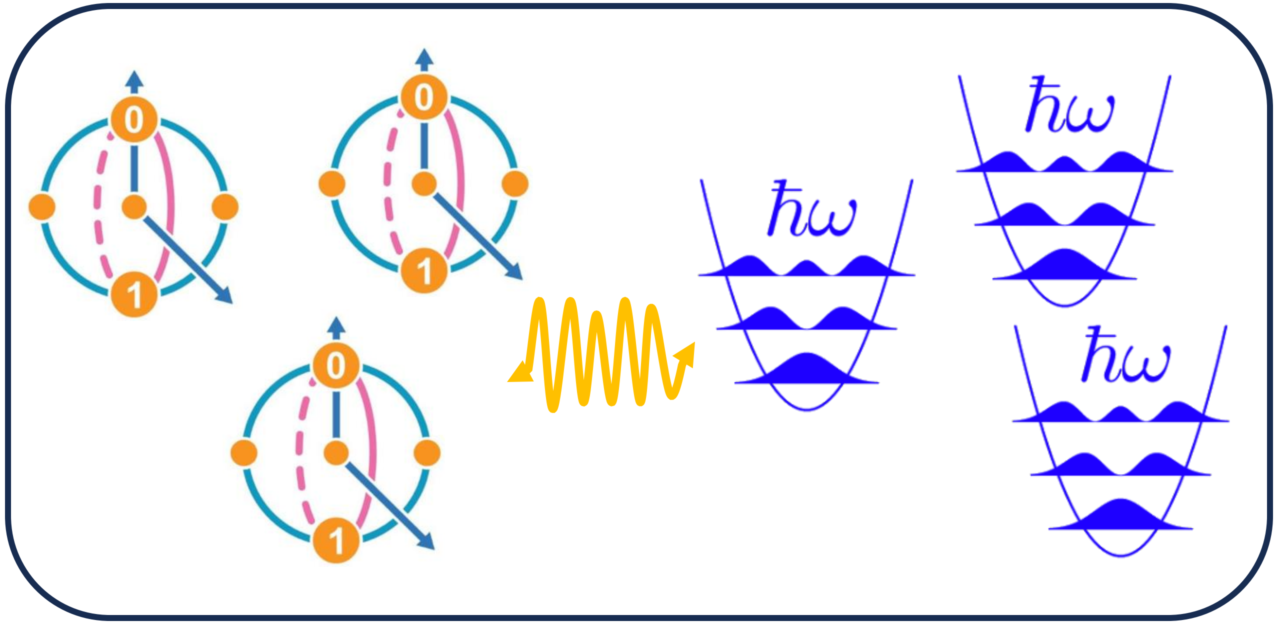
Evaluating the evidence for exponential quantum advantage in ground-state quantum chemistry

Allows quantitative resource estimation to assess quantum advantage.

Why is Making ISA and AMM Difficult for Hybrid Processors?

- Need a physical architecture
- Need a model for error correction
- Need a universal gate set
- Need effective compilation methods

A hybrid quantum computer



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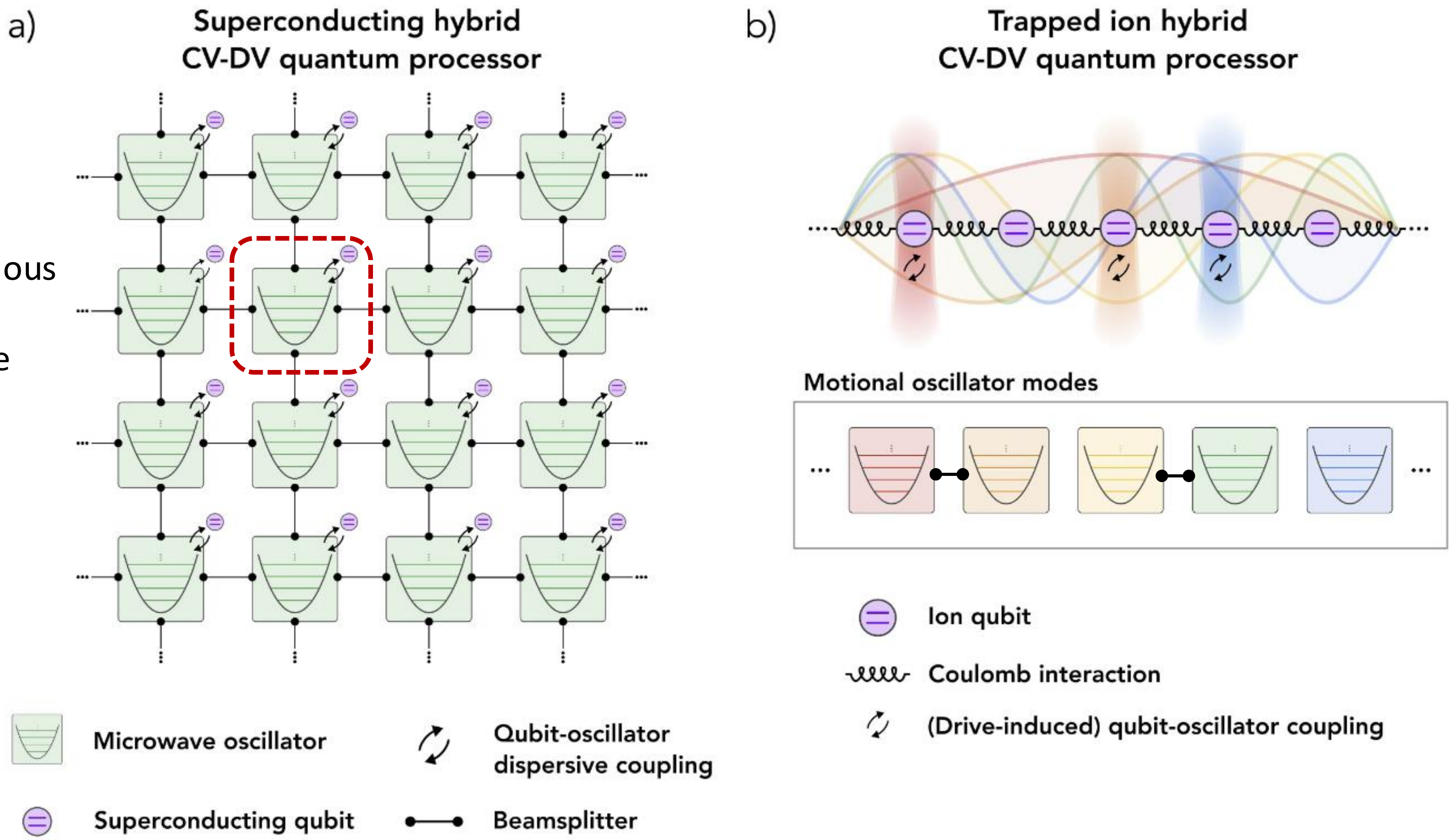
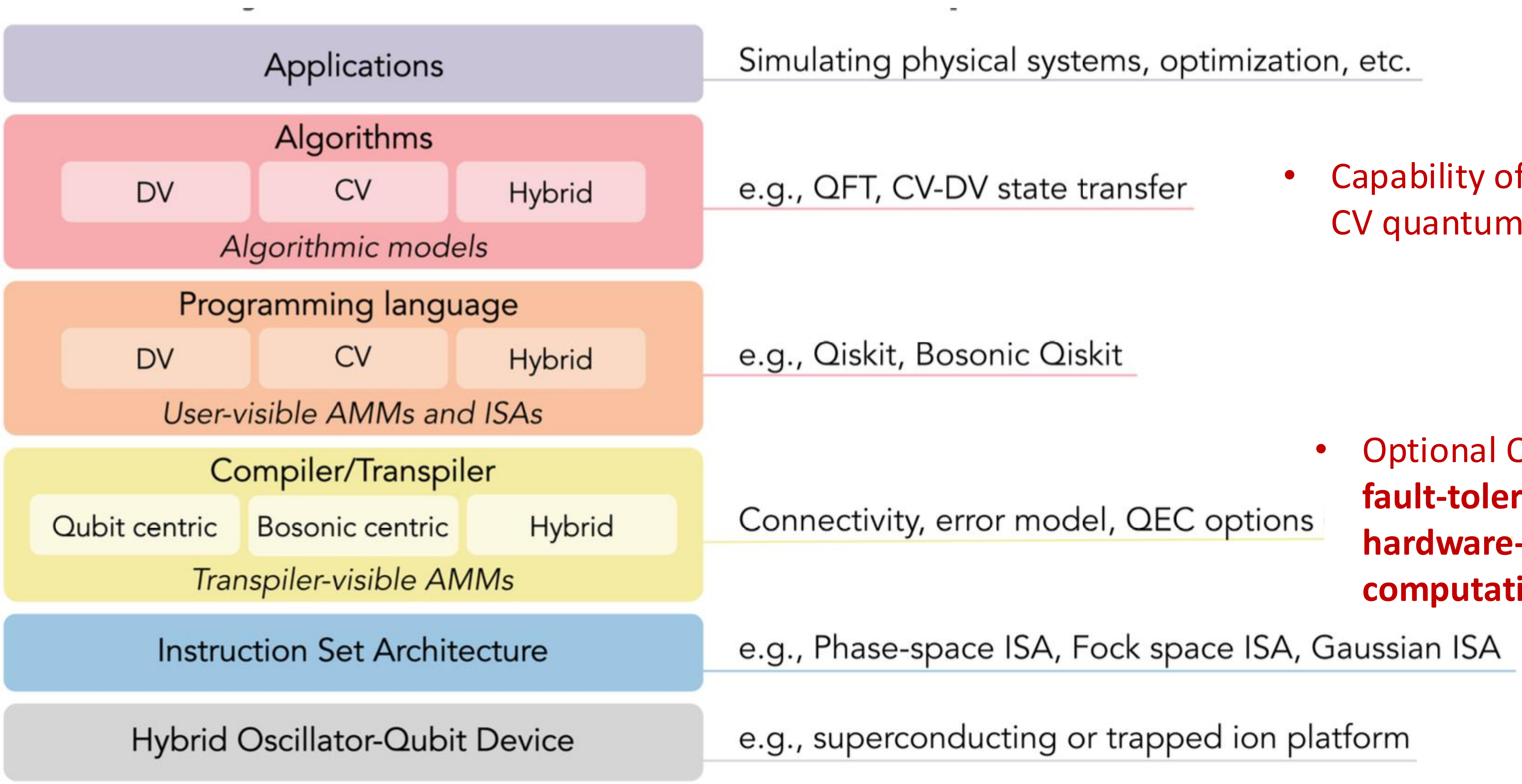
Physical Layout of a Hybrid Processor

CV: continuous variable
DV: discrete variable



Hybrid CV-DV quantum processor stack



- Capability of running **both** DV and CV quantum algorithms
- Optional QEC allows both **DV fault-tolerance** and **CV hardware-efficient computation**

Gates – The Key to Programmability

- Hybrid quantum processors allow us to define useful quantum gates.

Oscillator only gates:

$$\mathrm{SNAP}(\vec{\varphi}) = e^{-i\sum_n \varphi_n |n\rangle\langle n|}$$

$$\mathrm{SNAP}(\vec{\varphi})\sum_n \Psi_n|n\rangle = \sum_n e^{-i\varphi_n}\Psi_n|n\rangle,$$

- oscillator-only gates

Hybrid gates:

Conditional displacement

$$D_{\mathrm{c}}(\alpha,\beta) = |0\rangle\langle 0|\; D(\alpha) + |1\rangle\langle 1|\; D(\beta)$$

- hybrid entangling gates

Control flow: feedback, variational algorithms; modular architectures, distributed hybrid processor; autonomous error correction

Benchmarking: Certifying the gates are working as intended

Wigner function or process tomography via heterodyne detection

No randomized benchmarking

Quantifying complexity of measurement, computation, sampling, post-processing

Hybrid Universal Control

- **Qubit universal gate sets are well-known, while hybrid universal gate sets are less established.**

- Qubit universal control

$$X, Y, Z$$

- Oscillator universal control

$$C(r) = \exp\left(-ir\hat{x}^3\right)$$

$$H = \underbrace{h_0(\hat{x}, \hat{p})}_{\text{polynomial}}$$

- Hybrid universal control

$$G_{\mathrm{CD-ISA}} = \{X, Z, \hat{x}Z, \hat{p}Z\}$$

$$[\hat{x}Y, \hat{p}Z] = iX(\hat{x}\hat{p} + \hat{p}\hat{x})$$

$$[\hat{x}^n Y, \hat{p}^m Z] = i[\hat{x}^n\hat{p}^m + \hat{p}^m\hat{x}^n]X$$

$$H = \underline{h_0(\hat{x}, \hat{p})} + \underline{\vec{h}_1} \cdot \vec{\sigma}$$

Four independent polynomials

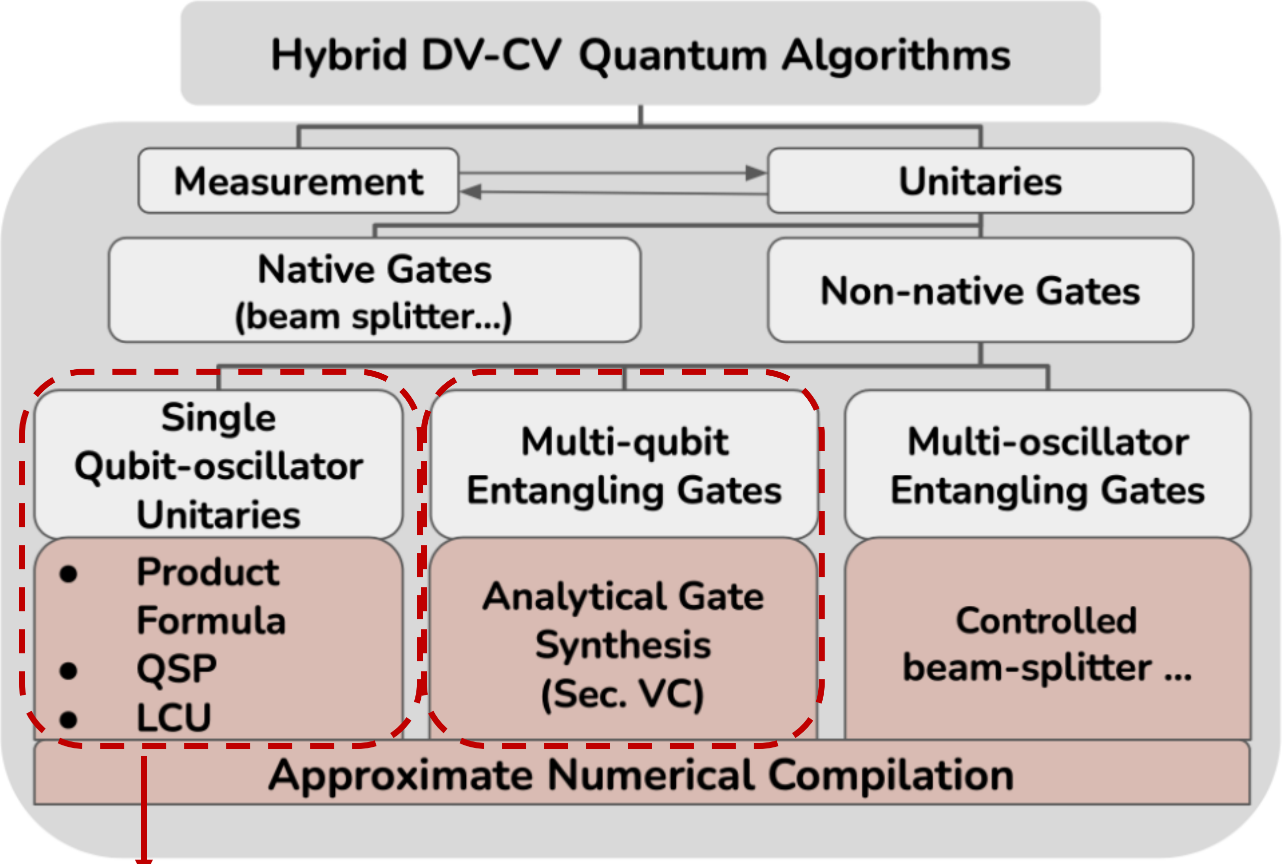
Universal Instruction Sets

	ISA Name	Minimum gate set
Linear oscillator control	Gaussian	$\mathcal{G} = \{D(\alpha), S(\zeta), \mathrm{BS}(\theta,\varphi) \text{ or } \mathrm{TMS}(r,\varphi)\}$
Universal oscillator control	Cubic	$\mathcal{G} + U_3(z)$
	Quartic	$\mathcal{G} + U_4(z)$
	SNAP	$\{D(\alpha), \mathrm{SNAP}(\vec{\varphi}), \mathrm{BS}(\theta,\varphi) \text{ or } \mathrm{TMS}(r,\varphi)\}$
Universal hybrid control	Phase-Space ISA	$\{\mathrm{CD}(\beta), R_\varphi(\theta), \mathrm{BS}(\theta,\varphi)\}$
	Fock-Space ISA	$\left\{\mathrm{SQR}(\vec{\theta},\vec{\varphi}), D(\alpha), \mathrm{BS}(\theta,\varphi)\right\}$
	Sideband ISA	$\{R_\varphi(\theta), \mathrm{JC}(\theta), \mathrm{BS}(\theta,\varphi)\}$

- Simple rotation, translation, squeeze of phase space distribution.
- Can achieve non-Gaussian operations on *m* oscillators!
- Can achieve fully universal control on an arbitrary set of *n* qubits and *m* oscillators!

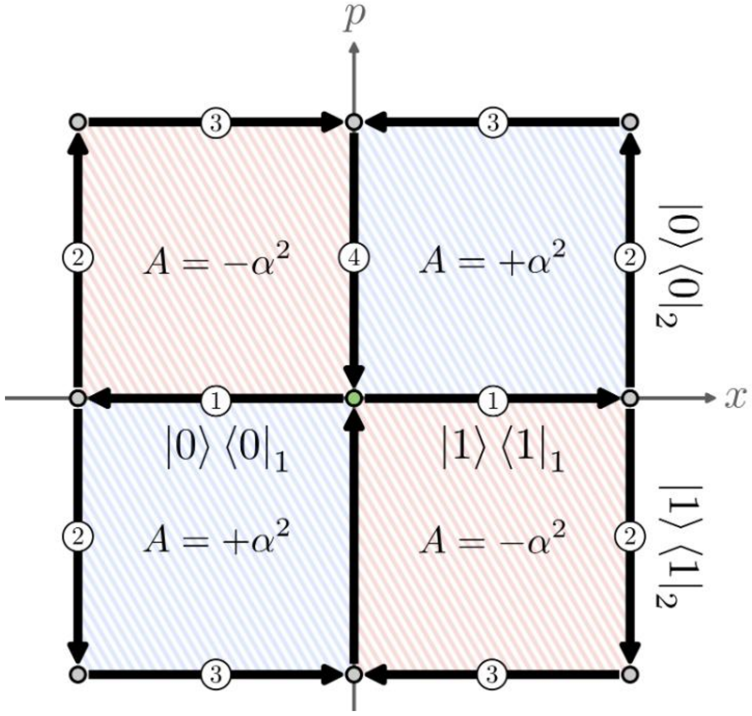
Compilation Methods

- **Full sets of analytical + numerical compilation methods are established for hybrid DV-CV quantum algorithms.**



Single qubit-oscillator unitaries are the key to unleash CV computational power!

Analytical compilation of multi-qubit entangling gates by leveraging geometric phase



$$R_{ZZ}(\theta) = e^{-i\frac{\theta}{2}Z_1 \otimes Z_2}$$

Kevin Smith, Brook Haven National Lab

4 CD gates,
4 BS gates.

Single qubit-oscillator compilation

- **Algorithmic primitives in DV quantum computation can be generalized to the hybrid CV-DV case.**

Hybrid CV-DV
Quantum Signal Processing (QSP)

Qubitization of bosonic mode:

w

$$e^{-i\frac{k}{2}\hat{x}\cdot\sigma_z} = \begin{bmatrix} \boxed{e^{-i\frac{k}{2}\hat{x}}} & \\ & e^{i\frac{k}{2}\hat{x}} \end{bmatrix} := W_z$$

v

$$e^{-i\frac{\lambda}{2}\hat{p}\cdot\sigma_z} = \begin{bmatrix} \boxed{e^{-i\frac{\lambda}{2}\hat{p}}} & \\ & e^{i\frac{\lambda}{2}\hat{p}} \end{bmatrix} \quad W_z^{(\lambda)}$$

- single-variable:

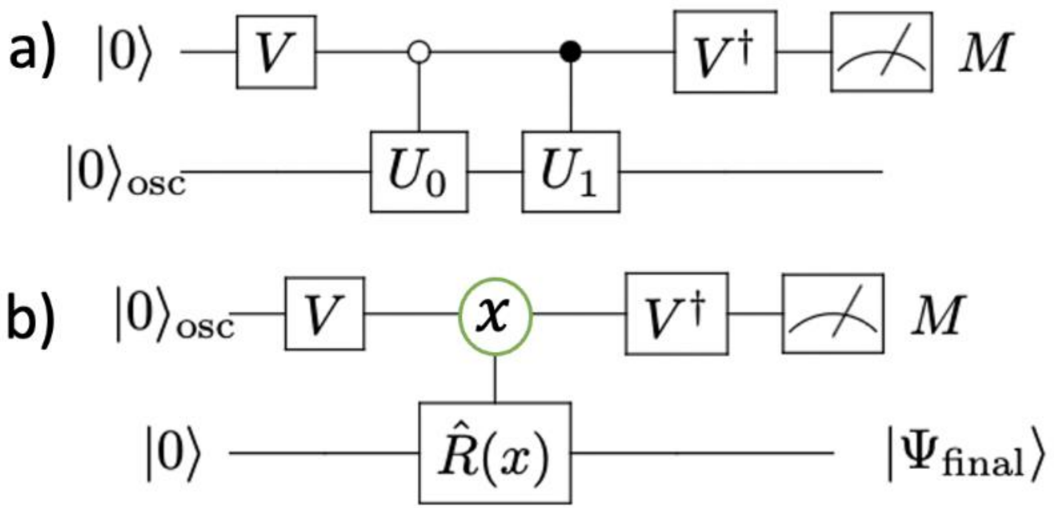
$$e^{i\phi_0\sigma_x}\prod_{j=1}^{d} W_z e^{i\phi_j\sigma_x} = \begin{bmatrix} F(w) & iG(w) \\ iG(w^{-1}) & F(w^{-1}) \end{bmatrix}$$

- Non-commutative bivariable (non-abelian):

$$e^{i\phi_0\sigma_x}\prod_{j=1}^{d} W_z^{(k)} e^{i\phi_j^{(k)}\sigma_x} W_z^{(\lambda)} e^{i\phi_j^{(\lambda)}\sigma_x} = \begin{bmatrix} F_d(w,v) & iG_d(w,v) \\ iG_d(v^{-1},w^{-1}) & F_d(v^{-1},w^{-1}) \end{bmatrix}$$

Non-linear transformation of oscillator quadrature operators!

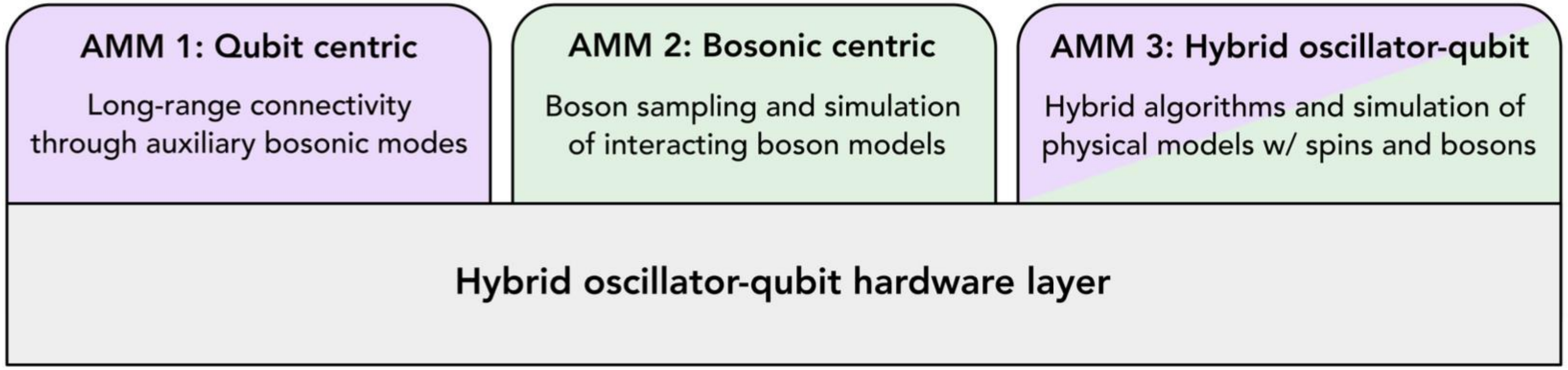
Linear Combination of Unitaries
(hybrid oscillator-qubit)

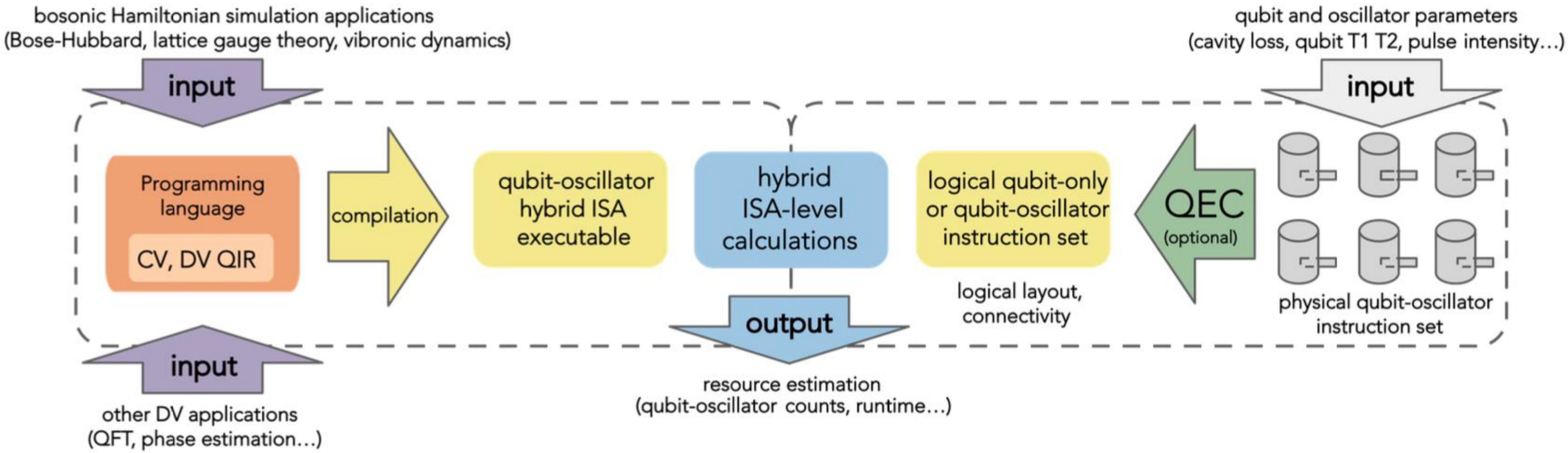


$$|\psi_{\text{final}}\rangle = \int_{-\infty}^{\infty} dx\; c_{x''}(x)\; \hat{R}(x)\,|0\rangle$$

where $c_{x''}(x) = \Psi(x)\,\langle x''|\,V^\dagger\,|x\rangle$

Resource Estimation on Hybrid Processors





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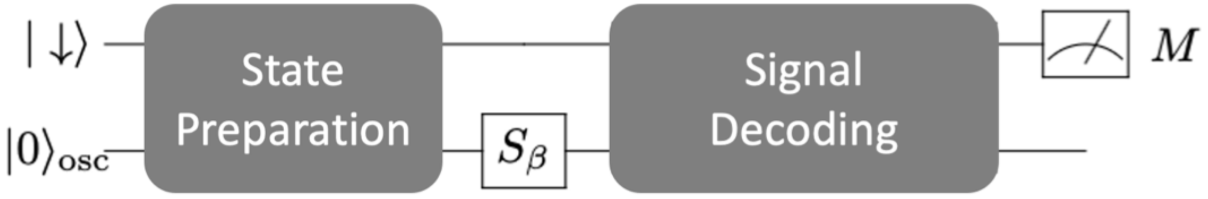
Example I: Quantum Sensing and Single-Shot Decision-Making

Designing new protocols for efficient single-shot decision-making is not easy.

The most general single-shot decision-making protocol

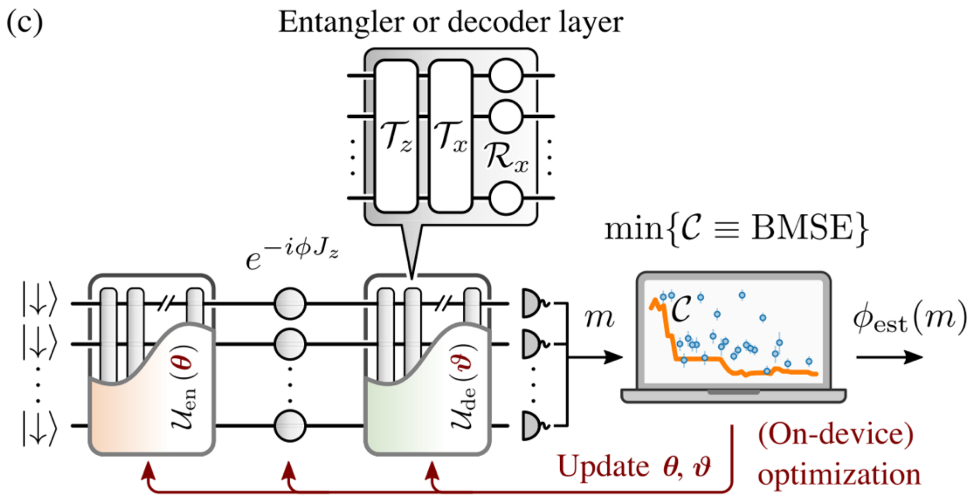


Decision = $\langle M \rangle = \underline{f}(\beta)$

Non-linear transformation

- General quantum sensing tasks can be reformulated as a problem of choosing proper form for $f(\cdot)$
- Just need to design the “State Preparation” and “Signal Decoding” unitaries.
- However, in general, we do not know how to construct the above unitaries that can realize a given $f(\cdot)$.

Variation algorithms for interferometer optimization



KAUBRUEGGER et al. PHYS. REV. X 11, 041045 (2021)

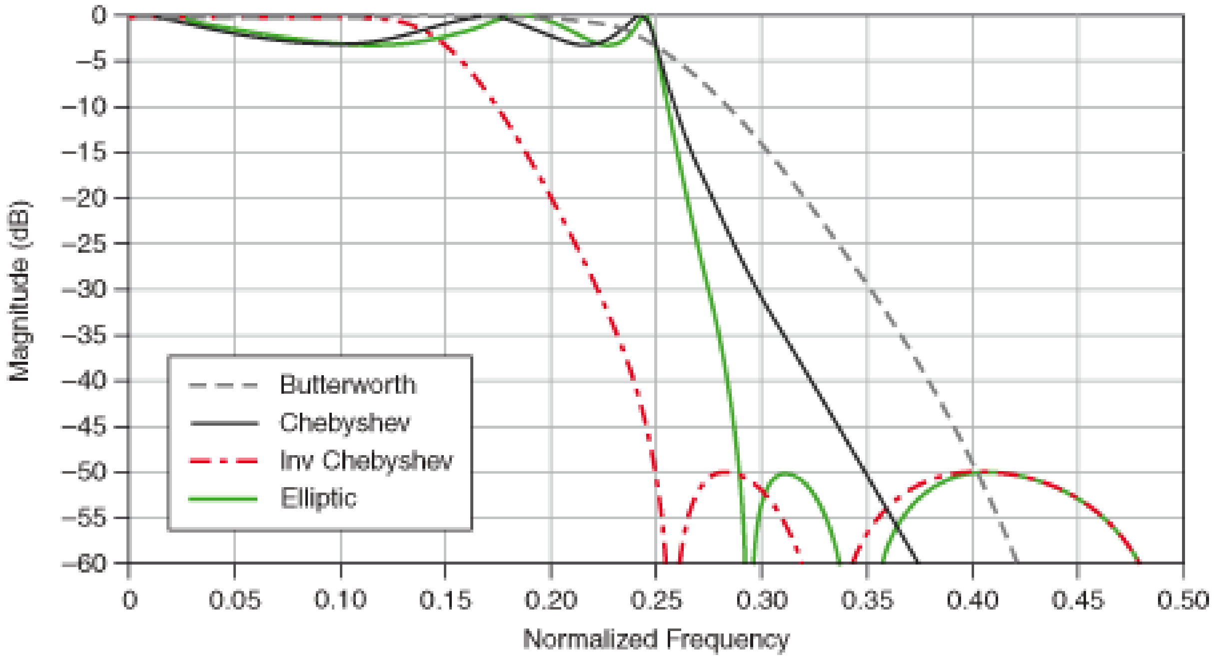
- Can use variational algorithms for state prep and signal decoding
- Difficult to show provable speedup

What can we learn from classical decision-making?

Decision = $\langle M \rangle = \underline{f}(\beta)$

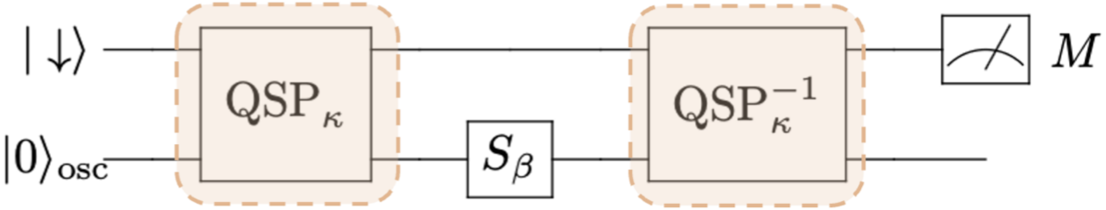
Non-linear transformation

- Classical signal processing (electrical engineering) routinely design *filters* (f) on *classical signals*:



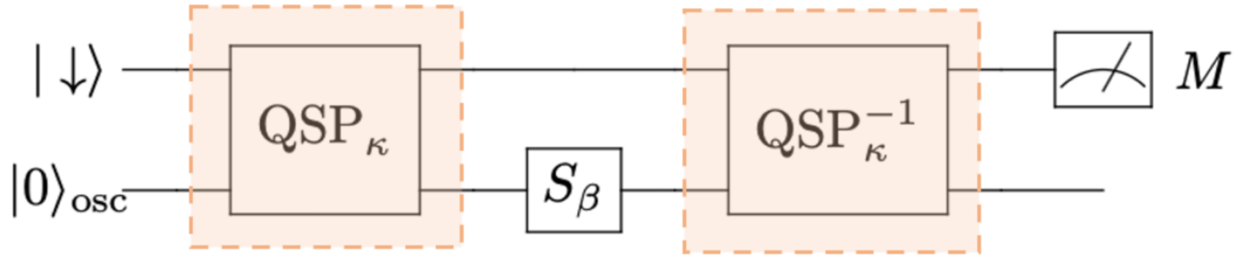
- We know how to perform signal processing on *quantum signals* (hybrid CV-DV quantum signal processing)!

Quantum signal processing interferometry (QSPI)



arXiv:2311.13703

QSPI for Displacement Sensing



Duality between two transformations

Polynomial transformation of the bosonic quadrature operators

Polynomial transformation of the sensing parameters!

$$\boxed{\mathbb{P}(M = \downarrow | \beta) = \sum_{s=-d}^{d} c_s v^s}$$

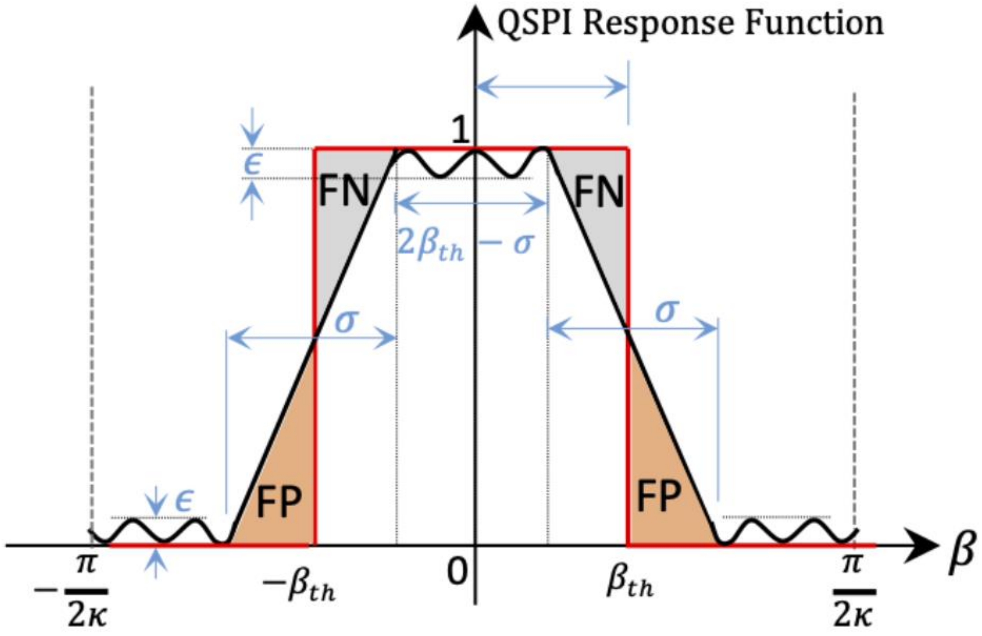
$$S(\beta) = e^{i\beta\hat{p}}$$

$$v = e^{i(2k)\beta}$$

What sensing tasks can this accomplish?

- Decision thresholding (step function)
- Band pass filter (window function)
- …

Qubit measurement probability vs. β



FN: false-negative error

FP: false-positive error

arXiv:2311.13703

Example II: Quantum Fourier Transform from Free-Evolution of an Oscillator

John Martyn MIT Physics
Jasmine Sinanan-Singh, MIT physics
Shraddha Singh, Yale Physics

Quantum Fourier Transform (*n* qubits)

$$|\psi\rangle_Q = \sum_{\mathbf{x}} c_{\mathbf{x}} |\mathbf{x}\rangle_Q$$

$$U_{\mathrm{QFT}}|\psi\rangle_Q = \sum_{\mathbf{x}} \left[\sum_{\mathbf{y}} \frac{1}{\sqrt{2^n}} c_{\mathbf{y}} e^{2\pi i x y/2^n} \right] |\mathbf{x}\rangle_Q$$

Oscillator free-evolution is a Fourier transform!

$$F = e^{-i\frac{\pi}{2}\hat{n}}$$

$$\hat{\boldsymbol{x}} \xrightarrow{} \hat{\boldsymbol{p}}$$

$\hat{n} = \hat{a}^\dagger \hat{a}$: number operator
$\hat{\boldsymbol{x}}$: position operator, $\hat{\boldsymbol{p}}$: momentum operator

Challenges

- Need CV-DV state transfer protocol

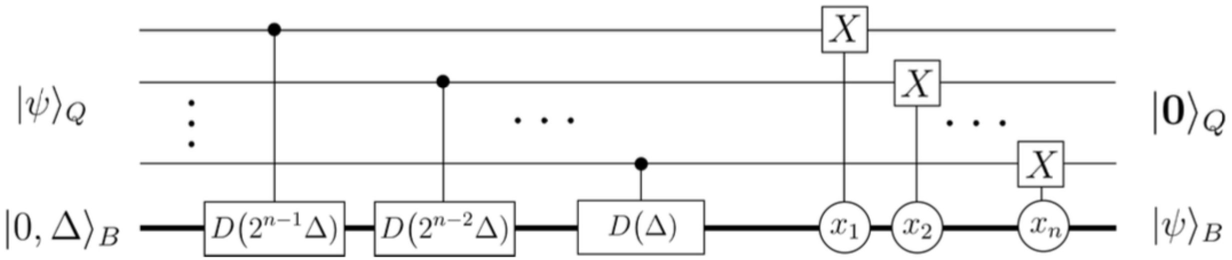
$$U_{\mathrm{st}}^{(n)}(\Delta)|\psi\rangle_Q|0,\Delta\rangle_B = |\mathbf{0}\rangle_Q|\psi\rangle_B$$

- **CV**: infinite dimensional, is **aperiodic**, defined in $(-\infty, +\infty)$
- **DV**: finite dimensional, **periodic**, 4π pulse drives $|0\rangle \rightarrow |1\rangle$ and back to $|0\rangle$ twice

- Is this efficient?

CV-DV state transfer protocol

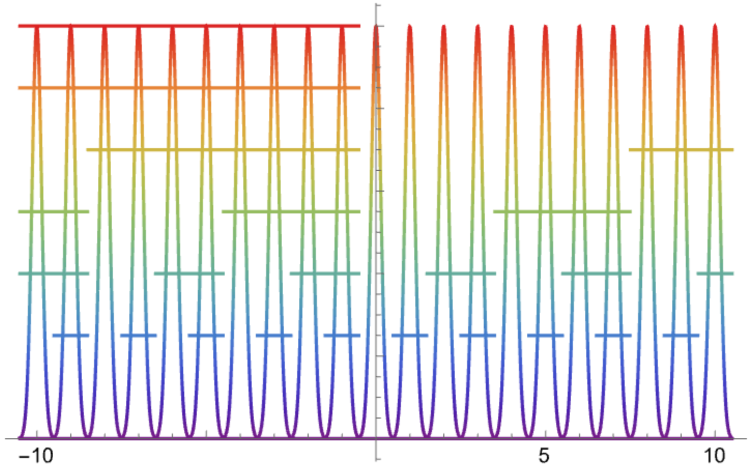
Single-variable hybrid CV-DV QSP



In preparation.

Fidelity: $1-\mathcal{O}(n\epsilon)-\mathcal{O}(e^{-\mathcal{O}(\Delta^2/\sigma^2)})$

Gate complexity: $\mathcal{O}(2^n\log(1/\epsilon))$



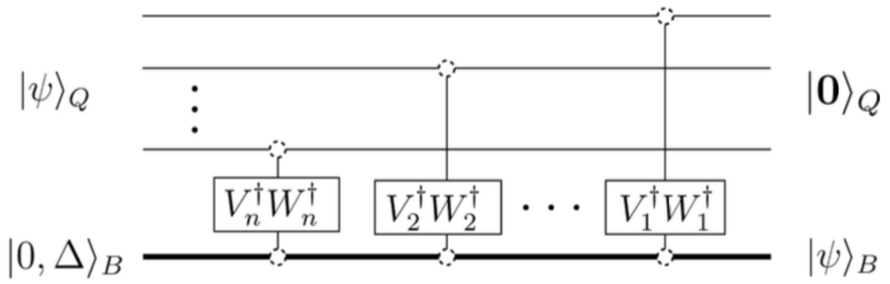
x

ϵ: error for QSP polynomials

σ: width of initial Gaussian

Δ: spacing between Gaussians

Non-abelian hybrid CV-DV QSP



$$1-\mathcal{O}\left(\int_{-\infty}^{-\lambda(2^n-1)} dq|\psi(q)|^2+\int_{\lambda(2^n-1)}^{\infty} dq|\psi(q)|^2\right)$$

$\mathcal{O}(n)$

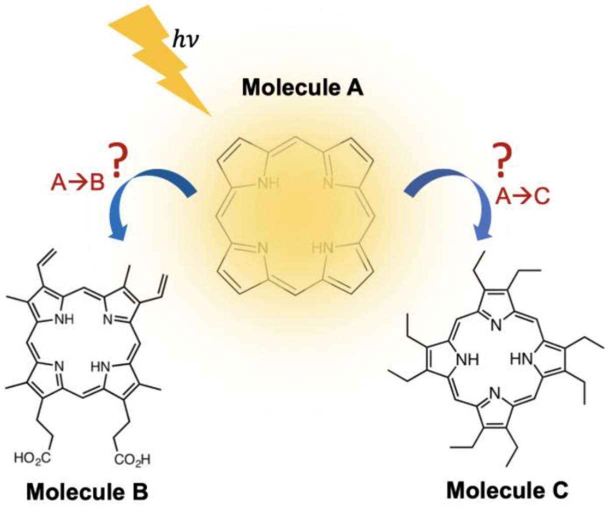
$$V_j = e^{i\frac{\pi}{2^{j+1}\lambda}\hat{x}\hat{\sigma}_y^{(j)}},\quad W_j = \begin{cases} e^{i\lambda 2^{j-1}\hat{p}\hat{\sigma}_x^{(j)}} & j<n,\\ e^{-i\lambda 2^{j-1}\hat{p}\hat{\sigma}_x^{(j)}} & j=n,\end{cases}$$

- Qubit QFT needs O(n^2)
- Caveat: may require a long time to implement, depending on experimental realizations such as strength of the driving fields.
- Quantum digital-analog sampling and interpolation!

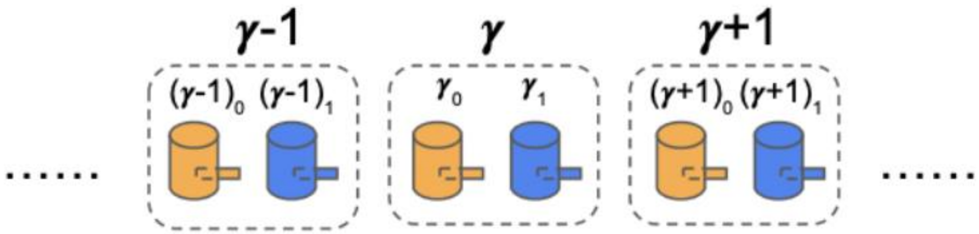
Kitaev and Webb, arXiv:0801.0342 (2008).
Hastrup et al., Phys. Rev. Lett. 128, 110503 (2022).

Example III: Resource estimation of photosynthesis on hybrid processors

Hamiltonian simulation of photosynthetic process (vibration + electronic)



- 1D hardware connectivity



N-chromophore model: $2N$ transmons + $2N$ cavity

$$H = \sum_{\gamma=1}^{N} H_0^{(\gamma)} + H_1^{(\gamma)} + H_2^{(\gamma)}$$

$$H_0^{(\gamma)} = \omega_{\gamma_0}\gamma_0^\dagger\gamma_0 + \omega_{\gamma_1}\gamma_1^\dagger\gamma_1 - \frac{\omega_{q\gamma_0}}{2}Z_{\gamma_0}$$

$$H_1^{(\gamma)} = -\frac{\chi_{\gamma_0}}{2}\gamma_0^\dagger\gamma_0 Z_{\gamma_0} - \frac{g_{cd,\gamma_0}}{2}(\gamma_0 + \gamma_0^\dagger)Z_{\gamma_0}$$ (Intra-chromophore interaction)

$H_2^{(\gamma)}$: interactions (<u>non-Condon</u>) between chromophores

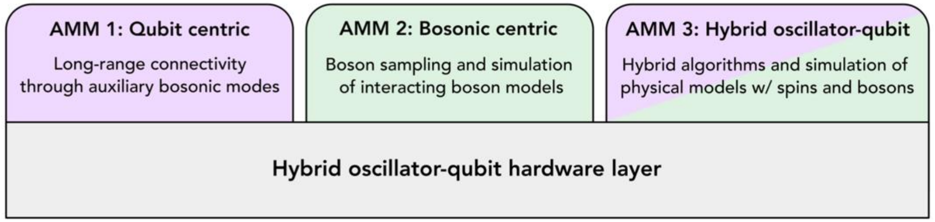
- Simulating 0.01 pico-second of the physical dynamics:

One Trotter step = $N \times (360\ BS + 368\ CD + 3\ SNAP)$

In preparation.

Conclusion

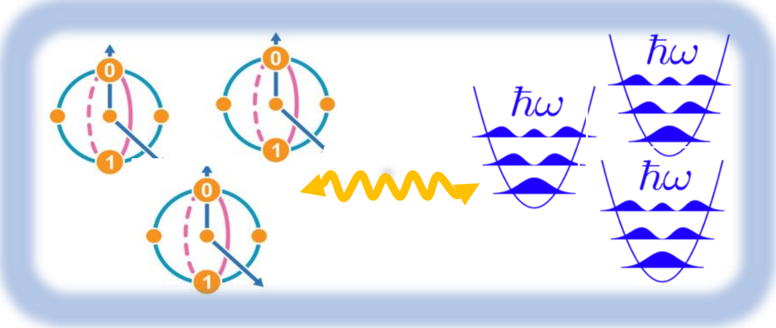
Abstract Machine Models (AMMs)



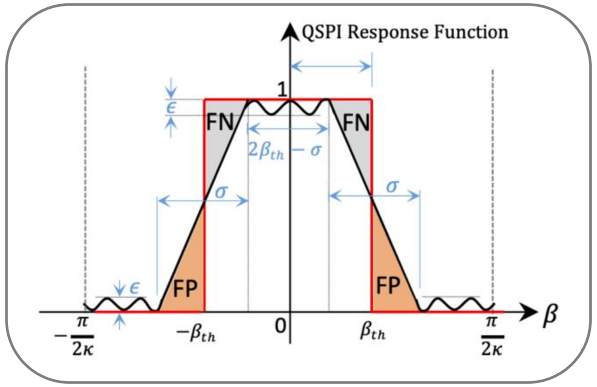
Instruction Set and Compilation

Universal hybrid control	Phase-Space ISA	$\{\mathrm{CD}(\beta), R_\varphi(\theta), \mathrm{BS}(\theta,\varphi)\}$
	Fock-Space ISA	$\left\{\mathrm{SQR}(\vec{\theta},\vec{\varphi}), D(\alpha), \mathrm{BS}(\theta,\varphi)\right\}$
	Sideband ISA	$\{R_\varphi(\theta), \mathrm{JC}(\theta), \mathrm{BS}(\theta,\varphi)\}$

Hybrid quantum processor



Quantum sensing

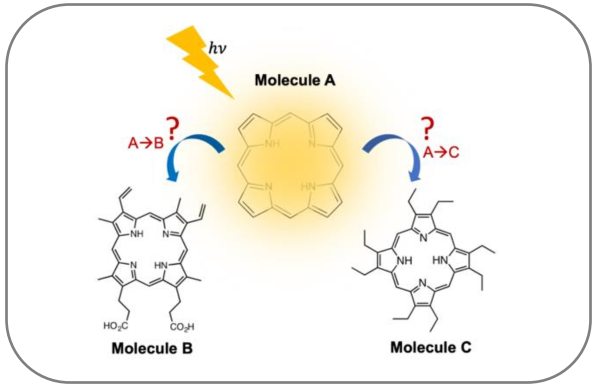


Quantum Fourier Transform

$$F = e^{-i\frac{\pi}{2}\hat{n}}$$

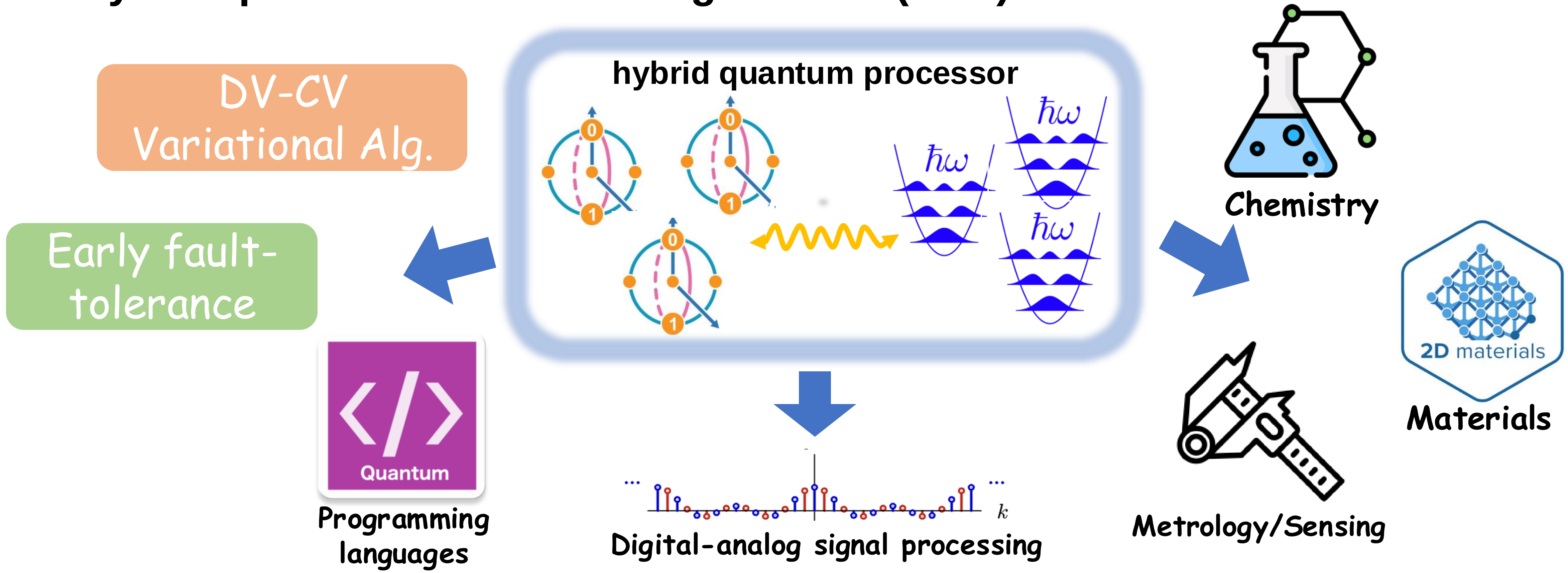
$$\hat{x} \longrightarrow \hat{p}$$

Quantum Chemistry



What is Next?

- **Compilation:** Complete theory of non-Abelian QSP; more ISAs
- **Architecture:** parallelism (instruction, processor); stack memory, QRAM; memory hierarchy
- **Hybrid quantum Arithmetic Logical Unit (ALU)**



NC STATE UNIVERSITY Electrical and Computer Engineering

NC STATE UNIVERSITY COMPUTER SCIENCE

- Quantum Engineering and Simulation Theory (QuEST)
    - quantum algorithms and simulation
    - hybrid continuous-discrete-variable QIP
    - quantum engineering
- Email: yliu335@ncsu.edu

Thanks!

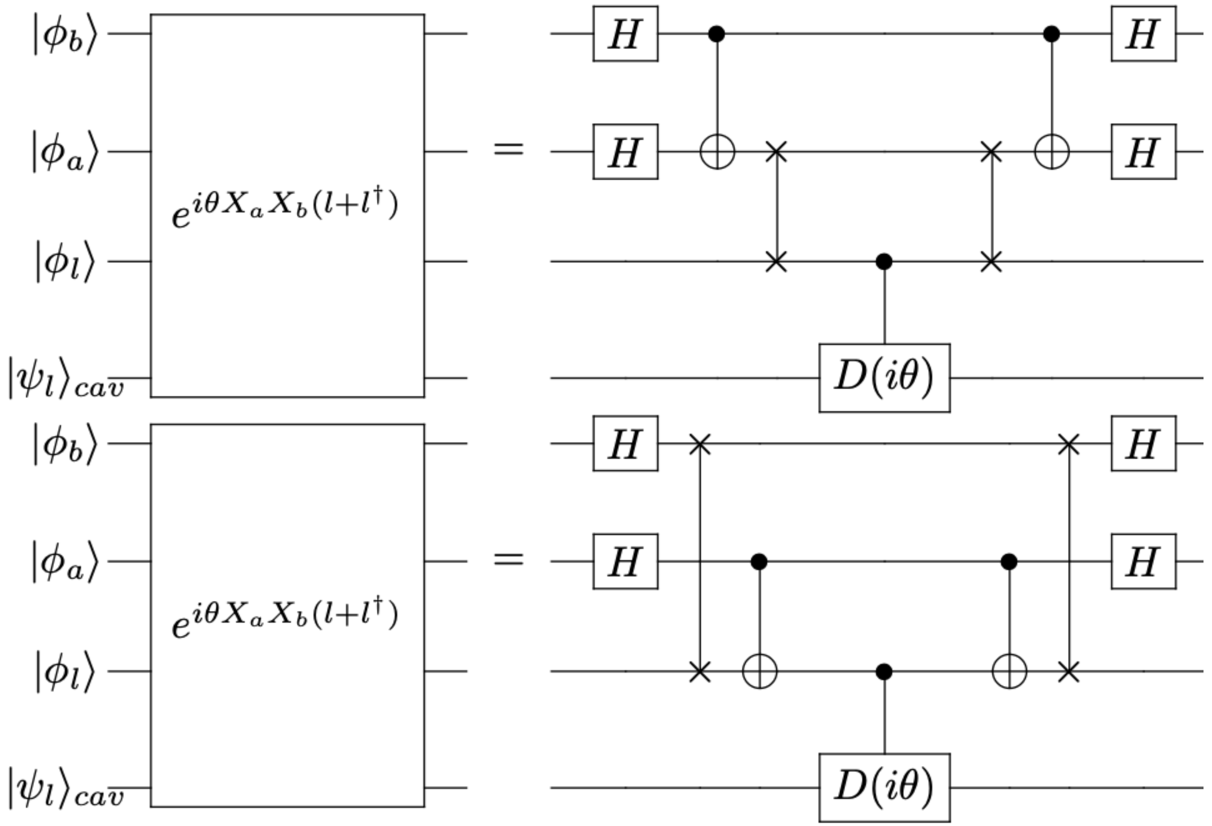
Non-Condon Hamiltonian for a 1D Chromophore Chain

$$H = \sum_{\gamma=1}^{N} H_0^{(\gamma)} + H_1^{(\gamma)} + H_2^{(\gamma)}$$

$$H_0^{(\gamma)} = \omega_{\gamma_0}\gamma_0^\dagger\gamma_0 + \omega_{\gamma_1}\gamma_1^\dagger\gamma_1 - \frac{\omega_{q\gamma_0}}{2}Z_{\gamma_0}$$

$$H_1^{(\gamma)} = -\frac{\chi_{\gamma_0}}{2}\gamma_0^\dagger\gamma_0 Z_{\gamma_0} - \frac{g_{cd,\gamma_0}}{2}(\gamma_0+\gamma_0^\dagger)Z_{\gamma_0}$$

$$\begin{aligned} H_2^{(\gamma)} = &- g_{cd,\gamma_1}(\gamma_1+\gamma_1^\dagger)\frac{Z_{\gamma_0}}{2} \\ &+ \frac{g_{\gamma_0,(\gamma-1)_0}}{2}(\sigma^+_{\gamma_0}\sigma^-_{(\gamma-1)_0} + h.c.) \\ &+ \frac{g_{\gamma_0,(\gamma+1)_0}}{2}(\sigma^+_{\gamma_0}\sigma^-_{(\gamma+1)_0} + h.c.) \\ &+ \frac{g_{\gamma_0,(\gamma-1)_0,\gamma_1}}{2}(\sigma^+_{\gamma_0}\sigma^-_{(\gamma-1)_0} + h.c.)(\gamma_1+\gamma_1^\dagger) \\ &+ \frac{g_{\gamma_0,(\gamma+1)_0,\gamma_1}}{2}(\sigma^+_{\gamma_0}\sigma^-_{(\gamma+1)_0} + h.c.)(\gamma_1+\gamma_1^\dagger) \end{aligned}$$



Quantum Sensing – The need to go beyond parameter estimation

- **Quantum sensing**: leveraging quantum entanglement supposition to improve sensing capability

- **Parameter estimation:** the standard deviation for the parameter of interest:

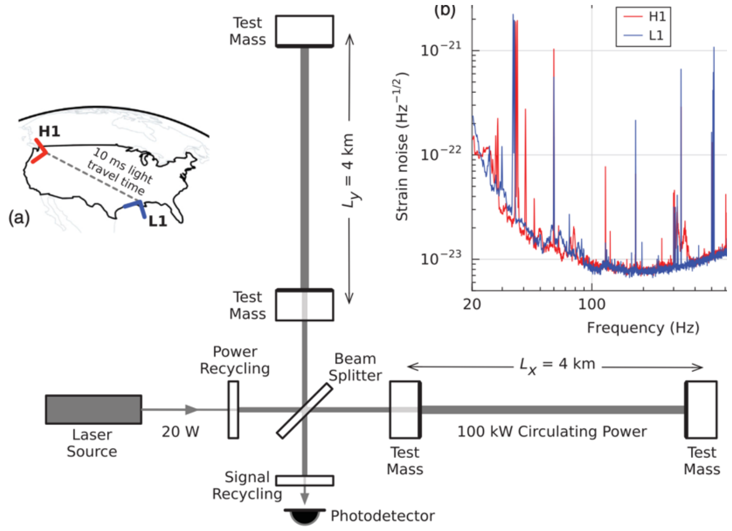
$$\frac{1}{\sqrt{N}} \rightarrow \frac{1}{N}$$ (N: time, number of probes, etc.)

Central limit theorem (shot-noise behavior)

Heisenberg limit

Bollinger et al., Phys. Rev. A 54, 4649 (1996)

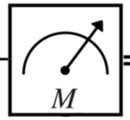
- Rare events need single-shot decision-making, e.g., gravitational wave detection:



- From estimation to decision?
- Why not directly perform decision-making using quantum protocols?

$\langle x \rangle, \Delta x$ →

U(x)

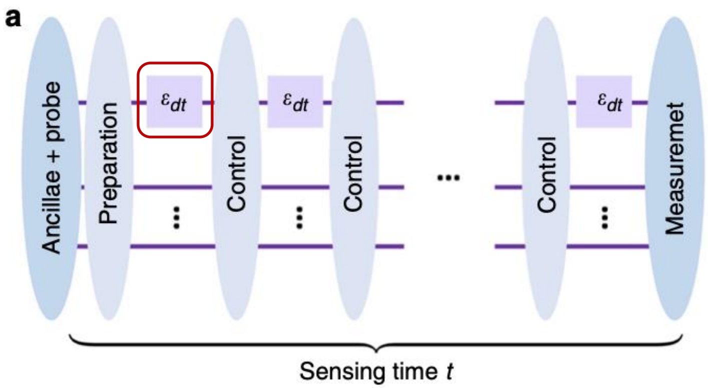


0 or 1?

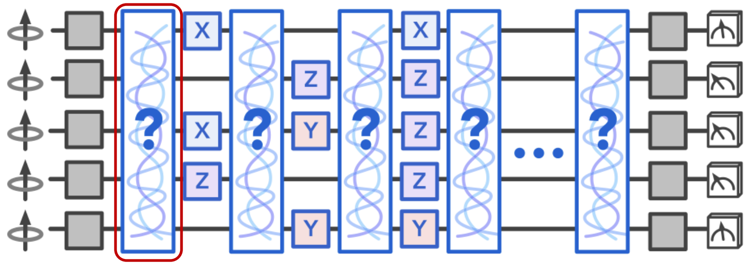
Why is it challenging to perform single-shot decision-making?

- Protocols for general sensing tasks beyond parameter estimation are rare, especially on bosonic modes.
  - *<u>Easy</u>: only one bit of information is needed*
  - *<u>Challenging</u>: global information about entire phase space is needed*

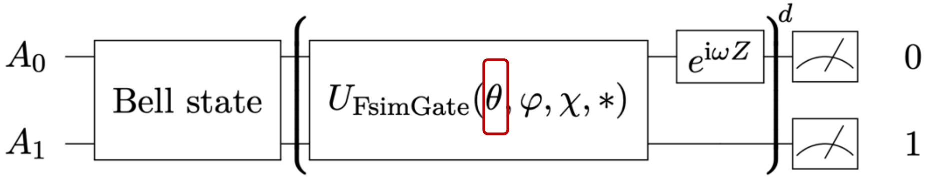
- Existing iterative protocols are challenged in the single-shot limit



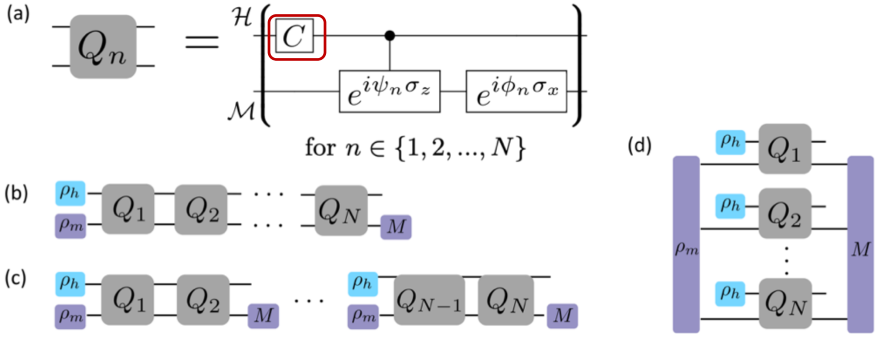
Zhou et al., *Nature Comm.* 9.1 (2018): 78.



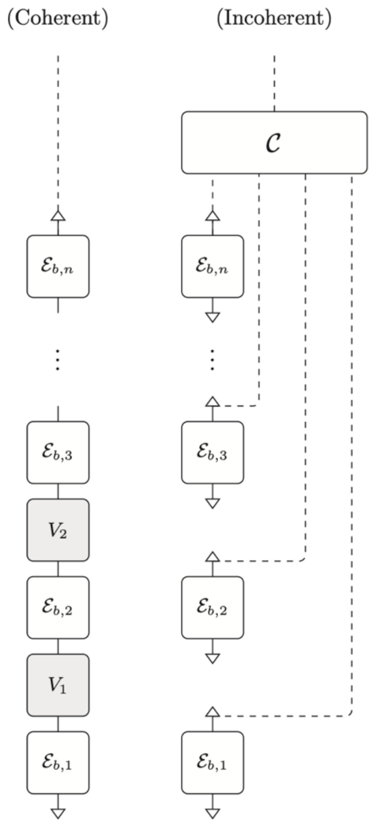
Huang et al., Phys. Rev. Lett. 130, 200403 (2023)



Dong et al., *arXiv:2209.11207* (2022).



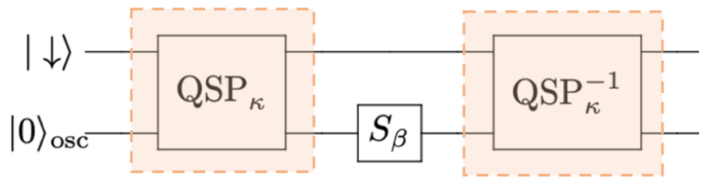
Sugiura et al., arXiv:2304.02053 (2023).



Rossi et al., Phys. Rev. A **105**, 032401 (2022)

Quantifying Binary Decision Quality

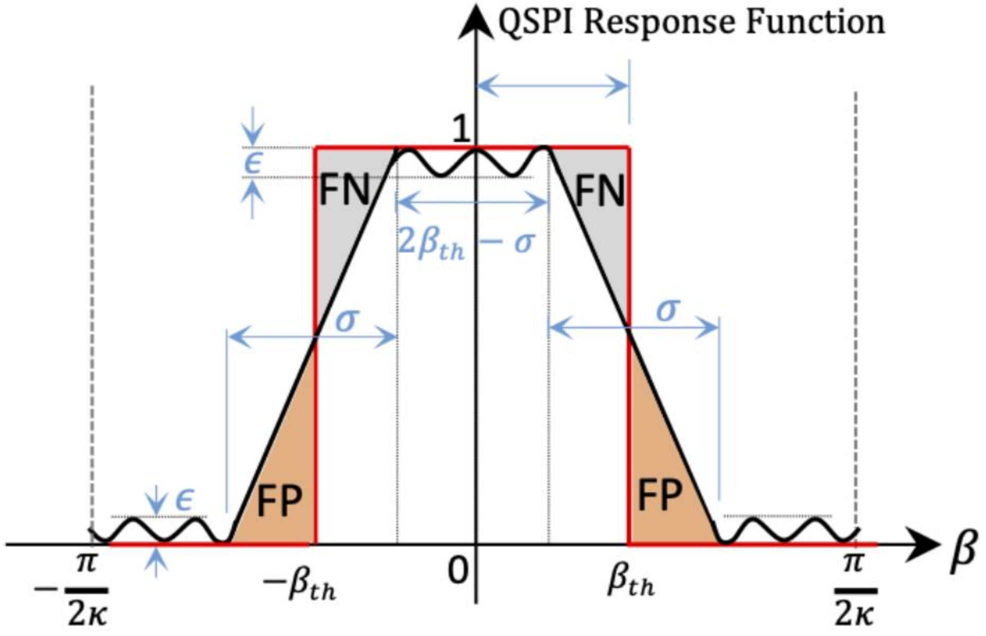
Nonlinear transformation of the sensing parameter in single-shot limit.



QSPI Response Function(β)

- ✓ Step function will be ideal (red)
- ✓ Actual qubit response function (black)
- ✓ Minimize error decision probability (shaded)

Qubit measurement probability vs. $\boldsymbol{\beta}$



FN: false-negative error

FP: false-positive error

How to quantify decision quality:

$$p_{\text{err}}(\beta_{\text{th}}, k) = \frac{k}{\pi} \int_{-\frac{\pi}{2k}}^{\frac{\pi}{2k}} |P_{\text{approx}}(\beta) - P_{\text{ideal}}(\beta)| d\beta$$

$$= p_{\text{err,FN}}(\beta_{\text{th}}) + p_{\text{err,FP}}(\beta_{\text{th}}).$$

Heisenberg scaling on decision error (single shot)

$$p_{\text{err}} \propto \frac{1}{kd} \log(d)$$

d is the circuit depth

Applications - Quantum Simulation

Eleanor Crane
(MIT Postdoc)

- State preparation; Hamiltonian simulation; observable measurement

Rabi Hamiltonian

$$H = \hbar\omega_m a^\dagger a + \hbar\omega_b \sigma^z + \frac{2\lambda}{\sqrt{N}}\left(a + a^\dagger\right)\sigma^x$$

Bose Hubbard

$$H = -J \sum_{\langle ij|ij\rangle} \left(b_i^\dagger b_j + \mathrm{h.c.}\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \mu \sum_i n_i$$

Lattice Gauge Theory

$$\begin{aligned} H &= H_0 + H_1 \\ H_0 &= -J_0 \sum_i a_i^\dagger Z_i a_{i+1} + \mathrm{h.c.} \\ H_1 &= -J_1 \sum_i X_i, \end{aligned}$$

Spin Systems

$$\begin{aligned} S &= [a^\dagger a + b^\dagger b]/2 = N/2 \\ S^z &= [a^\dagger a - b^\dagger b]/2 = [n_a - n_b]/2 \\ S^x &= [a^\dagger b + a b^\dagger]/2 \\ S^y &= -i[a^\dagger b - a b^\dagger]/2. \end{aligned}$$

Fermionic Matter
(quantum chemistry, materials)

$$f_p^\dagger f_q = \sigma_p^+ ZZ \ldots Z \sigma_q^-$$

$$f_p^\dagger f_q\, f_r^\dagger f_s = \cdots$$

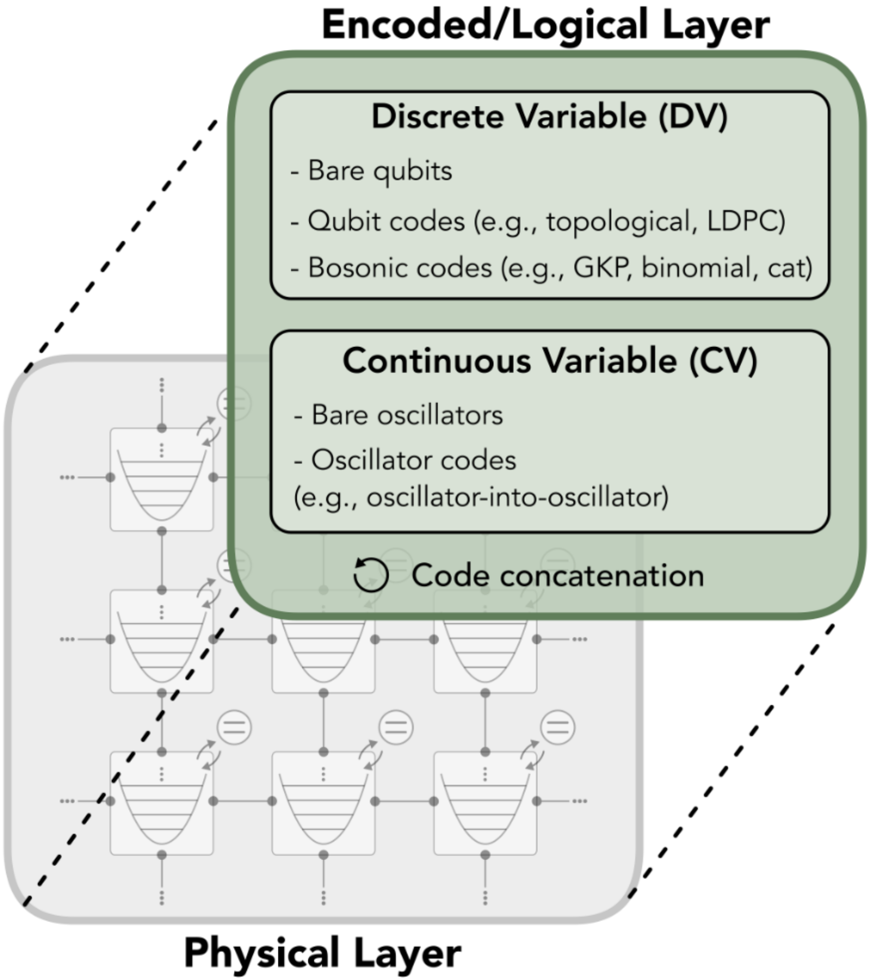
- Rigorous resource estimation
- Assessing quantum advantage

- **classical vs. quantum**
- **qubit-only vs. qubit-oscillator**

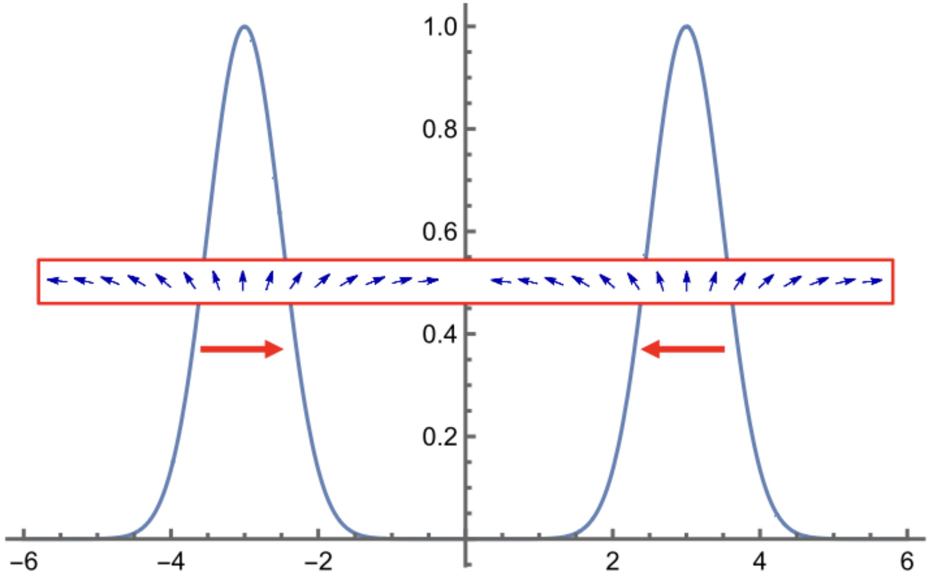
C^{2}QA

QEC and Logical ISA

Optional QEC



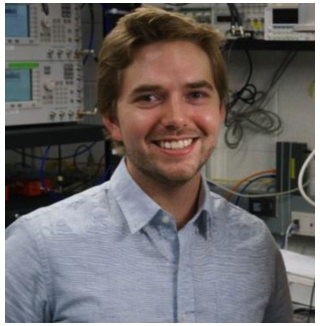
ISA for Cat state prep.



$$e^{-i\hat{x}\otimes\left(\frac{\pi}{4\alpha}\sigma_Y\right)}$$

$$e^{-i\left(\frac{\pi}{4\alpha}\hat{x}\right)\otimes\sigma_Y}$$

Shraddha Singh (Yale graduate student)



Alec Eickbusch (Yale graduate student)