Tutorial: Hybrid Oscillator-Qubit Quantum Processors:

Instruction Set Architectures, Abstract Machine Models, and Applications

[arXiv: 2407.10381]

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What is this tutorial about?

We know qubits are useful but what about oscillators?
[Superconducting circuits, trapped ions, Rydberg atom arrays, Photons]

1) Can we take advantage of their large Hilbert space?

2) Do they offer hardware efficiency in various tasks including quantum simulation?

3) Can we do quantum error correction with bosonic modes?

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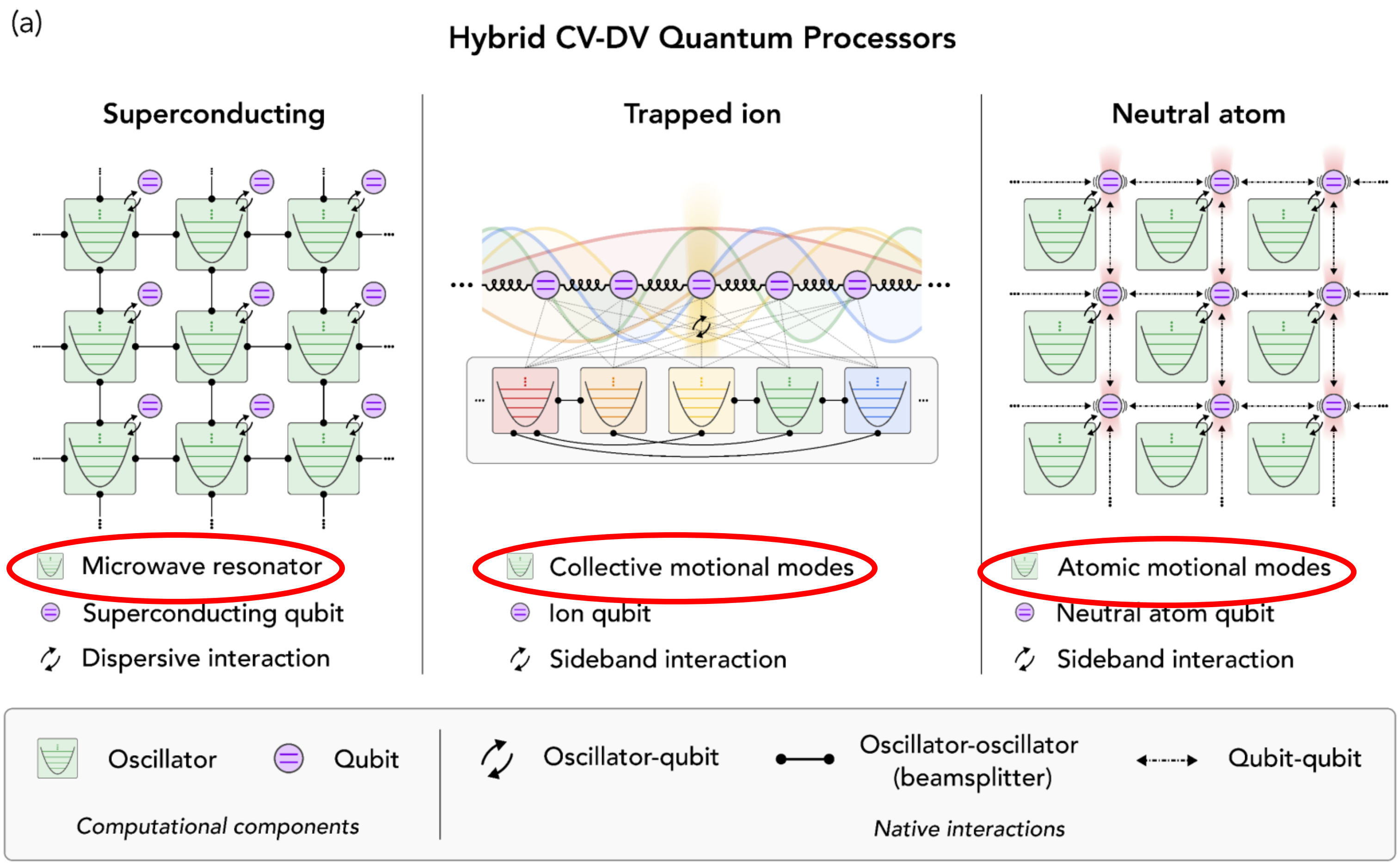
1) Can we take advantage of their large Hilbert space? **YES**

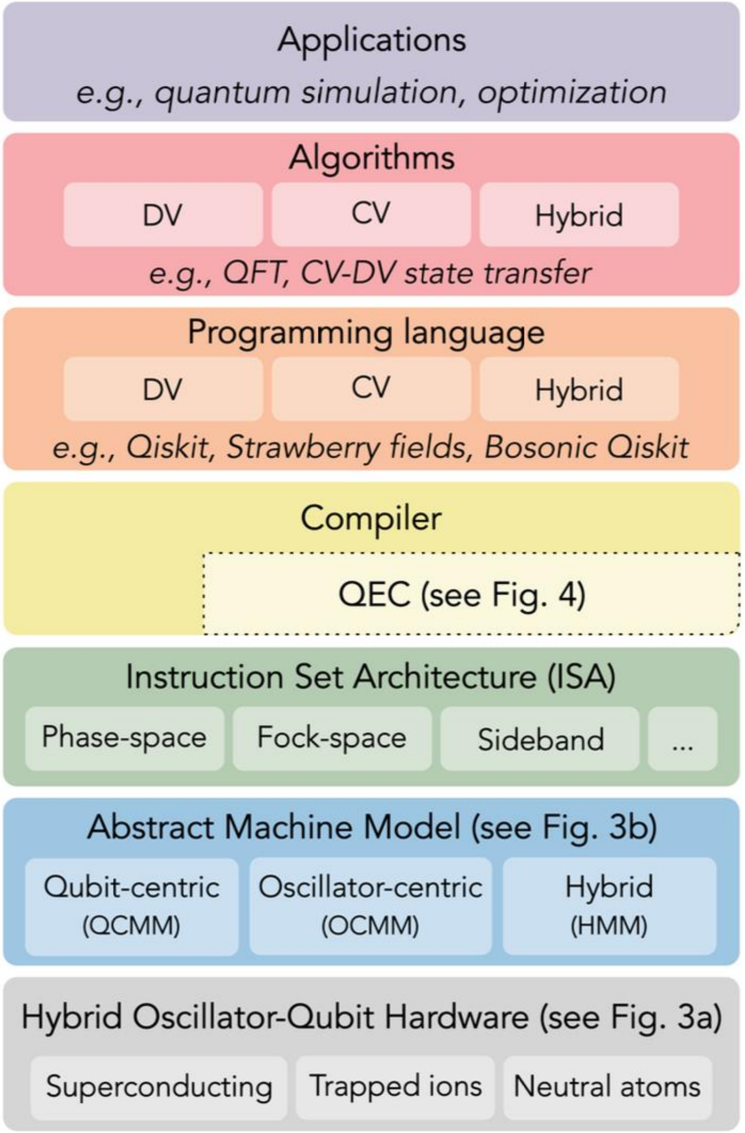
2) Do they offer hardware efficiency in various tasks including quantum simulation? **YES**

3) Can we do quantum error correction with bosonic modes? **YES**



CV: continuous-variable
DV: discrete-variable

Hybrid Oscillator-Qubit Quantum Computer Architecture



PART I - Physics and foundations of hybrid CV-DV quantum computation

- Continuous-variable (CV) states, operators, representations
- Universal CV-DV quantum computation
- Gaussian (Clifford), non-Gaussian (magic), and hybrid CV-DV gates
- Measurement
- Experimental realizations: superconducting and trapped ions

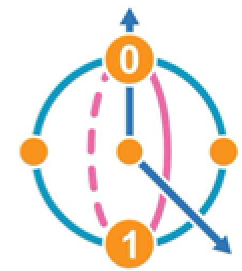
Continuous-Variable Quantum Computation 101

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m\omega_0^2 x^2$$

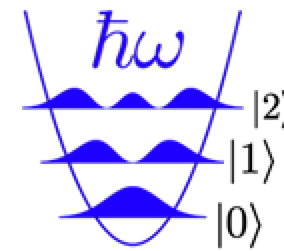
Cool it down to very low temperature
"quantize"

$$\hat{H}_0 = \omega_0(\hat{n} + \tfrac{1}{2})$$

Qubit



Q. Oscillator



Bosonic Quantum States

$$|0\rangle = [1 \quad 0 \quad 0 \quad \cdots]^T$$
$$|1\rangle = [0 \quad 1 \quad 0 \quad \cdots]^T$$
$$|2\rangle = [0 \quad 0 \quad 1 \quad 0 \quad \cdots]^T$$

$$|\psi\rangle = \textstyle\sum_{n=0}^{\infty} c_n |n\rangle$$

$$c_n \in \mathbb{C}$$

$$\hat{p} = \frac{i(a^\dagger - a)}{\sqrt{2}}$$

$$\hat{x} = \frac{a + a^\dagger}{\sqrt{2}}$$

Bosonic Quantum Gates

$$|\psi'\rangle = \underline{e^{-ih(\hat{x},\hat{p})}}|\psi\rangle$$

Single-oscillator gate

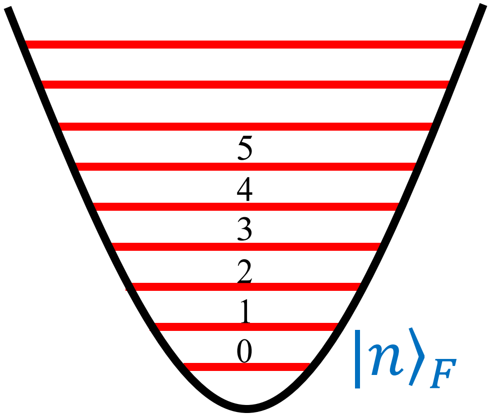
+ gates to Entangle two oscillators

$$\hat{x} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 1 & 0 & \sqrt{2} & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \cdots \\ 0 & 0 & \sqrt{3} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad \hat{p} = \frac{i}{\sqrt{2}}\begin{bmatrix} 0 & -1 & 0 & 0 & \cdots \\ 1 & 0 & -\sqrt{2} & 0 & \cdots \\ 0 & \sqrt{2} & 0 & -\sqrt{3} & \cdots \\ 0 & 0 & \sqrt{3} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$h(\hat{x},\hat{p})$ is a function of $\hat{x}, \hat{p}$

Recall: $R_{\hat{\mathbf{b}}}(\theta) = e^{-i\frac{\theta}{2}\hat{\mathbf{b}}\cdot\boldsymbol{\sigma}}$ $\qquad \sigma_x^2 = 1$

Two views of the oscillator Hilbert space:



(a) Discrete but countably infinite Fock basis:

$$H = \omega \hat{n} = \omega a^{\dagger} a$$

$$|\Psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle + \psi_2 |2\rangle + \psi_3 |3\rangle + \dots$$

(b) Continuous position/momentum basis:

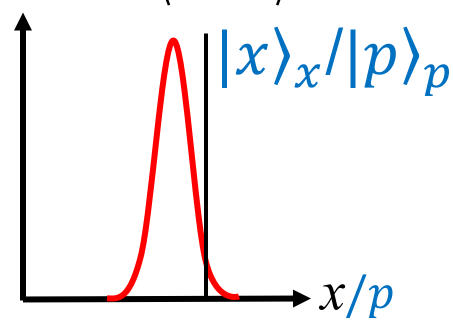
Annihilation/creation operator: $a = \hat{x} + i\hat{p} \qquad a^{\dagger} = \hat{x} - i\hat{p}$

Position/momentum operator: $\hat{x} \equiv \dfrac{a + a^{\dagger}}{2}, \quad \hat{p} = \dfrac{a - a^{\dagger}}{2i}$

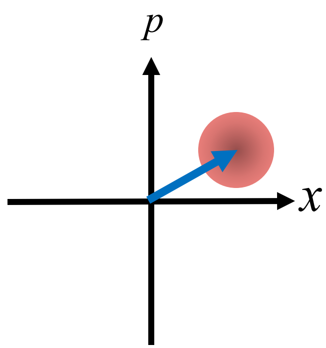
$[\hat{x}, \hat{p}] = \dfrac{i}{2}$ ['Wigner' units]

delta function $\delta(x - x') = \langle x | x' \rangle$

$\psi(x) = \langle x | \Psi \rangle$



Position



Phase Space

Hierarchy of Continuous-Variable (CV) States:

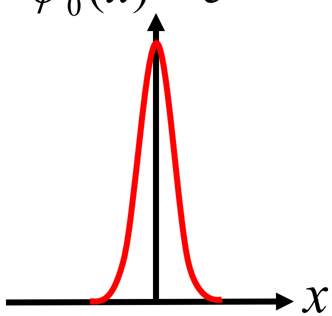
Level 0: Gaussian states

coherent state: $\left|\alpha\right\rangle = e^{\alpha a^\dagger - \alpha^* a}\left|0\right\rangle = e^{2i[\alpha_I\hat{x}-(\alpha_R\hat{p}]}\left|0\right\rangle$

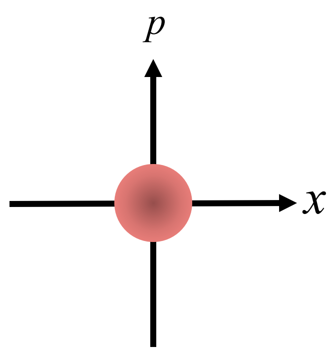
Eigenstate of *a*: $\left\langle\alpha\middle|a\middle|\alpha\right\rangle = \left\langle\alpha\middle|\hat{x}+i\hat{p}\middle|\alpha\right\rangle = \alpha_R + i\alpha_I = \alpha$

vacuum: $|\, n = 0\rangle$

$\psi_0(x) \sim e^{-x^2}$

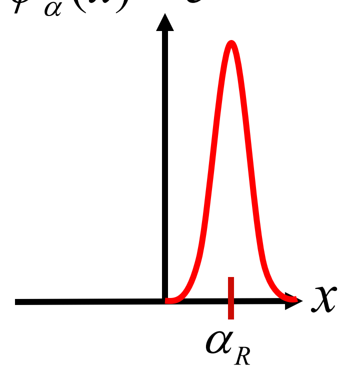


Position

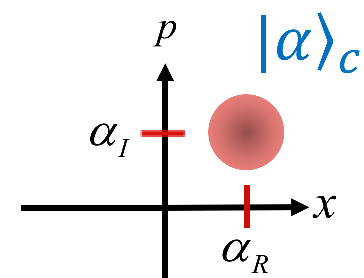


Phase Space

$\psi_\alpha(x) \sim e^{-(x-\alpha)^2}$



Position Space



Phase

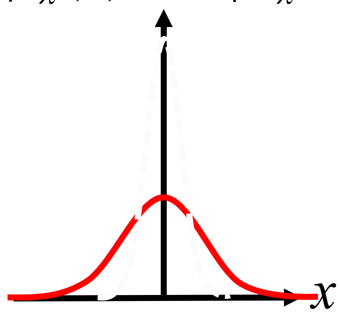
Vacuum state displaced in phase space

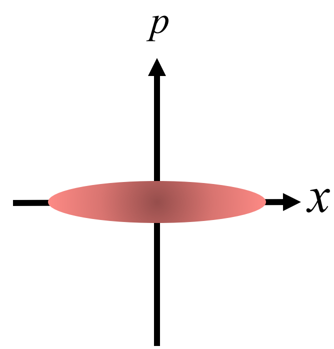
Hierarchy of Continuous-Variable (CV) States:

Level 0: Gaussian states

squeezed vacuum: $|0_\lambda\rangle = e^{-i\frac{\lambda}{2}(a^2 - a^{\dagger 2})}|0\rangle = e^{-i\lambda(\hat{x}\hat{p}+\hat{p}\hat{x})}|0\rangle$

$\psi_\lambda(x) = \langle x | 0_\lambda \rangle$

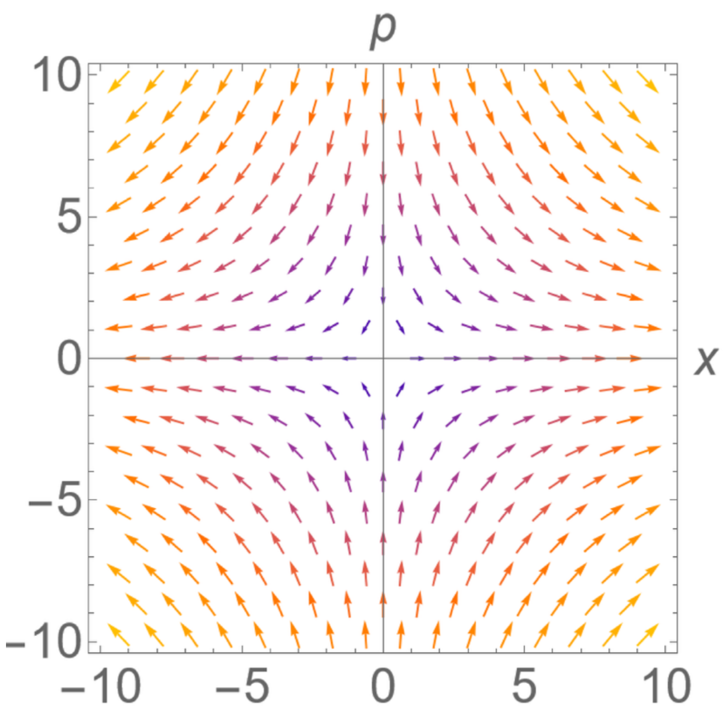


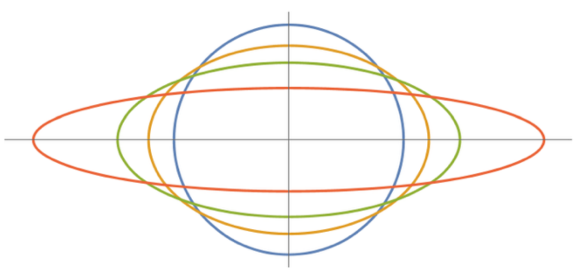


Position

Phase Space

(area preserving diffeomorphism)





Visualizing flows in phase space can be useful:

[Classical] squeezing Hamiltonian

$$H = px$$

$$\dot{x} = \frac{\partial H}{\partial p} = x \qquad x(t) = x(0)e^{t}$$

$$\dot{p} = -\frac{\partial H}{\partial x} = -p \qquad p(t) = p(0)e^{-t}$$

$$x(t)\,p(t) = \text{ constant}$$

Q: Is continuous-variable QC the same as qudit-based QC?

No, because CVs are technically infinite-dimension, only when truncate will be finite-dimension.

Q: What if we truncate the oscillator to d-levels?

No, because bosonic statistics can be missing.

$$a^\dagger|m\rangle = \underline{\sqrt{m+1}}|m+1\rangle$$

$$a|m+1\rangle = \underline{\sqrt{m+1}}|m\rangle$$

$$\left[a, a^\dagger\right] = 1$$

$$\hat{n} = a^\dagger a,$$

Sqrt(m) is hides the hardness / advantages.

Hierarchy of Oscillator Gate Operations
(view any unitary as Hamiltonian evolution)

$$U = e^{-i\, h_n(\hat{x},\hat{p})}$$

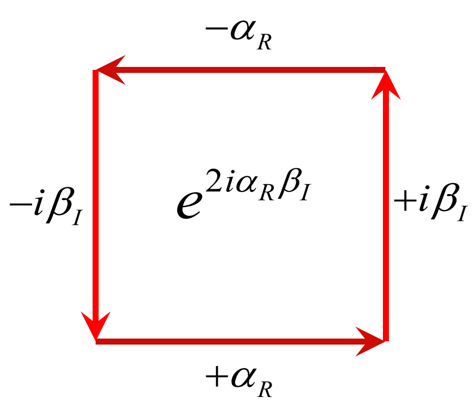
hermitian polynomial of degree n

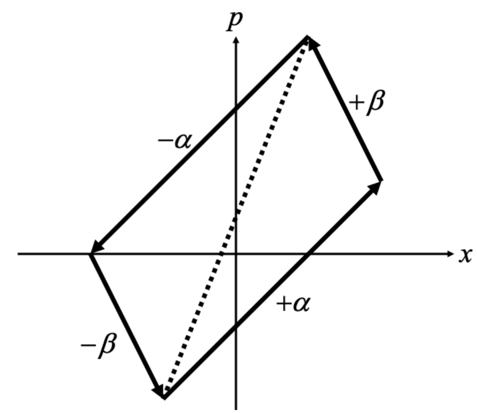
$$h_1(\hat{x},\hat{p}) = -2[\alpha_I \hat{x} - \alpha_R \hat{p}]$$

Generates displacements in phase space

$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a} = e^{2i[\alpha_I \hat{x} - \alpha_R \hat{p}]}$$

Phase space translations do not commute
Phase space has constant Berry curvature





Weyl-Heisenberg Group

$$D(i\beta_I)D(\alpha_R) = e^{+i\alpha_R\beta_I} D(\alpha_R + i\beta_I)$$
$$= e^{+2i\alpha_R\beta_I} D(\alpha_R)D(i\beta_I)$$

CV Analog to the DV Pauli Group

$$XY = e^{i\pi/2} Z$$
$$= e^{i\pi} YX$$

$$[\hat{x},\hat{p}] = \frac{i}{2} \qquad [a, a^\dagger] = 1$$

Proof:

Baker–Campbell–Hausdorff formula: $e^X e^Y = e^Z$

$$Z = X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] + \frac{1}{12}[Y,[Y,X]] + \cdots$$

Equivalently: $e^{B+C} = e^{-\frac{1}{2}[B,C]} e^B e^C$

Can motivates the study of CV stabilizer QEC codes, e.g., arXiv:2411.04993

Displacements are the CV analog of the Paulis.

What is the CV analog of the DV Clifford Group?

Recall:
DV Clifford operations C map the Pauli group to itself

$$CPC^{\dagger} = P'$$

What CV operations map displacements to displacements?

$$U_2 = e^{-i\, h_2(\hat{x},\hat{p})}$$

hermitian polynomial of degree 2

$$e^X Y e^{-X} = \sum_{n=0}^{\infty} \frac{[X,Y]_n}{n!}$$

$$[X,Y]_n \equiv \underbrace{[X,\cdots[X,[X,Y]]\cdots]}_{n \text{ times}}$$

$$[\hat{x},\hat{p}] = \frac{i}{2} \qquad [a, a^{\dagger}] = 1$$

$$h_2(\hat{x},\hat{p}) = \Big[\underbrace{\omega\left(\hat{x}^2+\hat{p}^2\right)}_{\text{rotation}} + \underbrace{R_1\left(\hat{x}^2-\hat{p}^2\right)}_{\text{squeezing}} + \underbrace{R_2\left(\hat{x}\hat{p}+\hat{p}\hat{x}\right)}_{\text{squeezing}}\Big]$$

Quadratic Hamiltonians generate phase space rotations and scale changes (squeezing)

$$\text{rotation: } \begin{pmatrix} \hat{x}' \\ \hat{p}' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{p} \end{pmatrix}$$

$$\text{squeezing: } \begin{pmatrix} \hat{x}' \\ \hat{p}' \end{pmatrix} = \begin{pmatrix} e^{r} & 0 \\ 0 & e^{-r} \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{p} \end{pmatrix}$$

Quadratic Hamiltonians generate maps of displacements to displacements

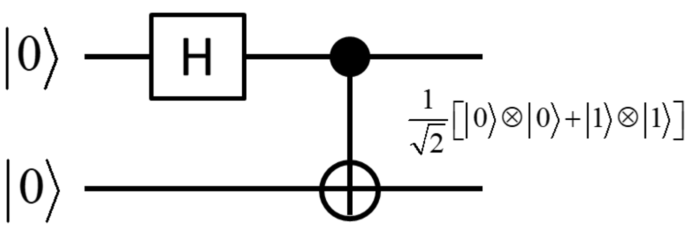
$$D(\alpha) = e^{\alpha a^{\dagger} - \alpha^* a} = e^{2i[\alpha_I \hat{x} - \alpha_R \hat{p}]}$$

$$U_2 D(\alpha) U_2^{\dagger} = e^{2i[\alpha_I \hat{x}' - \alpha_R \hat{p}']} = D(\alpha')$$

Exercise:
Show $[h_2, h_1] = h_1'$, $[h_2,[h_2, h_1]] = h_1''$, $[h_2, h_1]_n = h_1'''$

Quadratic Hamiltonians are CV analogs to generators of the Clifford group

2-qubit DV Clifford operations like CNOT can create entanglement:



CV two-mode squeezing generates entanglement:

$$h_2(\hat{x}_1,\hat{p}_1,\hat{x}_2,\hat{p}_2)=\frac{1}{2}(\hat{x}_1\hat{p}_2+\hat{p}_1\hat{x}_2)=\frac{i}{2}\left(a_1^\dagger a_2^\dagger-a_1a_2\right)$$

$$U_2|0\rangle\otimes|0\rangle\propto\sum_{n=0}^{\infty}e^{-rn}|n\rangle\otimes|n\rangle,\quad \text{(number correlated)}$$

Similar quadratic Hamiltonians can generate

$$U_2'|0\rangle\otimes|0\rangle\propto\int dx\,|x\rangle\otimes|x\rangle,\quad \text{(position correlated)}$$

How to see this?

Taylor expansion, and re-collecting all terms for fixed *n*.

DV Clifford circuits create entanglement but are easy to simulate classically by tracking stabilizer evolution

Non-Clifford DV gates (e.g., T gates) produce 'magic' (i.e., non-stabilizerness) needed to violate Bell inequalities and achieve quantum advantage.

CV 'Clifford' circuits map Gaussian states to Gaussian states but are easy to simulate classically by following evolution of mean and covariance matrix of the Gaussians.

Non-Clifford CV gates (e.g., generated by cubic Hamiltonians) produce 'non-Gaussianity' and 'Wigner negativity' required to achieve quantum advantage.

It is clear what 'non-Gaussianity' is. What about 'Wigner negativity'?

Deep Dive on Hybrid Gates:

Box IV.7. Conditional Displacement

$$D_c(\alpha, \beta) = |0\rangle\langle 0| \otimes D(\alpha) + |1\rangle\langle 1| \otimes D(\beta), \qquad (224)$$

$$\begin{aligned} \mathrm{CD}(\alpha) &= D_c(+\alpha, -\alpha) \\ &= \exp\left[\sigma_z \otimes \left(\alpha a^\dagger - \alpha^* a\right)\right] \end{aligned}$$

Why are these two definitions equivalent?

For real β

$$\mathrm{CD}(\beta) = e^{-2i\beta\sigma_z\hat{p}},$$
$$\mathrm{CD}(i\beta) = e^{+2i\beta\sigma_z\hat{x}}.$$

$$[\hat{x}, \hat{p}] = \frac{i}{2} \qquad \left[a, a^\dagger\right] = 1$$

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$$\begin{aligned} \mathrm{CD}(\alpha) &= e^{\sigma_z(\alpha a^\dagger - \alpha^* a)} \\ &= \sum_{k=0}^{\infty} \frac{1}{k!} [\sigma_z(\alpha a^\dagger - \alpha^* a)]^k \\ &= \sum_{k=even}^{\infty} \frac{1}{k!} I \otimes (\alpha a^\dagger - \alpha^* a)^k + \sum_{k=odd}^{\infty} \frac{1}{k!} \sigma_z \otimes (\alpha a^\dagger - \alpha^* a)^k \end{aligned}$$

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&= (|0\rangle\langle 0| + |1\rangle\langle 1|) \otimes \sum_{k=even}^{\infty} \frac{1}{k!}(\alpha a^\dagger - \alpha^* a)^k + (|0\rangle\langle 0| - |1\rangle\langle 1|) \sum_{k=odd}^{\infty} \frac{1}{k!}(\alpha a^\dagger - \alpha^* a)^k\\
&= |0\rangle\langle 0| \otimes \sum_{k=0}^{\infty} \frac{1}{k!}(\alpha a^\dagger - \alpha^* a)^k + |1\rangle\langle 1| \otimes \sum_{k=0}^{\infty} \frac{1}{k!}(-\alpha a^\dagger + \alpha^* a)^k\\
&= |0\rangle\langle 0| \otimes D(\alpha) + |1\rangle\langle 1| \otimes D(-\alpha)\end{aligned}$$

$-(\alpha)^k = (-\alpha)^k$ for odd k

How does displacement gate work?

$$D^\dagger(\alpha) = D(-\alpha)$$
$$D^\dagger(\alpha) a D(\alpha) = a + \alpha.$$

Note:

$$a \;\; = \hat{x} + i\hat{p}$$
$$a^\dagger = \hat{x} - i\hat{p},$$

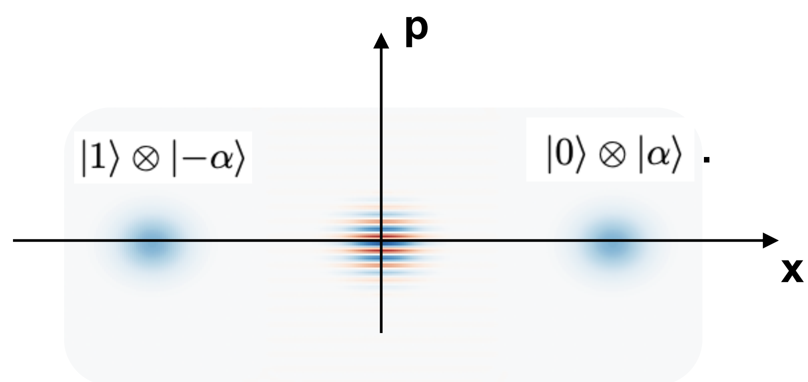
This means displacement shifts the position by Re(α), and shift momentum by Im(α).

$$e^X Y e^{-X} = \sum_{n=0}^{\infty} \frac{[X, Y]_n}{n!}$$

$$[X, Y]_n \equiv \underbrace{[X, \cdots [X, [X, Y]] \cdots]}_{n \text{ times}}$$

$$\hat{I} = \frac{1}{\pi} \int d^2\alpha \; |\alpha\rangle\langle\alpha| = \sum_{n=0}^{\infty} |n\rangle\langle n|$$

$$\mathrm{CD}(\alpha) \, |+\rangle \otimes |0\rangle_F$$



Box IV.2. Phase-Space Rotation Gate

$$R(\theta) = e^{-i\theta\hat{n}}. \tag{148}$$

Quantum Fourier Transform Gate

$$F = R\left(\frac{\pi}{2}\right) = e^{-i\frac{\pi}{2}\hat{n}}. \tag{149}$$

$$F^\dagger \hat{x} F = +\hat{p} \tag{150}$$

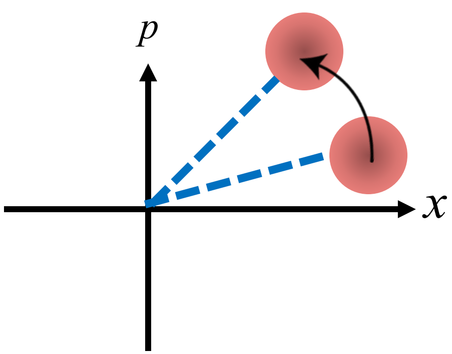
$$F^\dagger \hat{p} F = -\hat{x}. \tag{151}$$

$$e^X Y e^{-X} = \sum_{n=0}^{\infty} \frac{[X,Y]_n}{n!}$$

$$[X,Y]_n \equiv \underbrace{[X,\cdots[X,[X,Y]]\cdots]}_{n \text{ times}}$$

$$\begin{aligned}
R(\theta)|\alpha\rangle &= |e^{-i\theta}\alpha\rangle, \\
R^\dagger(\theta) &= R(-\theta), \\
R^\dagger(\theta) a R(\theta) &= e^{-i\theta} a, \\
R^\dagger(\theta) a^\dagger R(\theta) &= e^{+i\theta} a^\dagger, \\
R^\dagger(\theta) \hat{x} R(\theta) &= \cos\theta\,\hat{x} + \sin\theta\,\hat{p}, \\
R^\dagger(\theta) \hat{p} R(\theta) &= \cos\theta\,\hat{p} - \sin\theta\,\hat{x},
\end{aligned}$$

$$\begin{aligned}
\langle \hat{x}' \rangle &= \langle\psi|\, R^\dagger \hat{x} R \,|\psi\rangle \\
&= \langle\psi|\cos\theta\ \hat{x} + \sin\theta\ \hat{p}\,|\psi\rangle \\
&= \cos\theta\ \langle\hat{x}\rangle + \sin\theta\ \langle\hat{p}\rangle
\end{aligned}$$



Analog of the SWAP on oscillators:

Box IV.4. Beam-splitter Gate

$$\mathrm{BS}(\theta,\varphi) = e^{-i\frac{\theta}{2}\left[e^{i\varphi}a^\dagger b + e^{-i\varphi}ab^\dagger\right]}. \tag{169}$$

In the interaction picture, the beam-splitter gate transforms the two bosonic mode operators as follows

$$a_{\theta,\varphi} = \mathrm{BS}^\dagger(\theta,\varphi)\, a\, \mathrm{BS}(\theta,\varphi) = \cos\frac{\theta}{2}a - i\sin\frac{\theta}{2}e^{+i\varphi}b \tag{170}$$

$$b_{\theta,\varphi} = \mathrm{BS}^\dagger(\theta,\varphi)\, b\, \mathrm{BS}(\theta,\varphi) = \cos\frac{\theta}{2}b - i\sin\frac{\theta}{2}e^{-i\varphi}a. \tag{171}$$

$$[\mathrm{BS}(\theta,\varphi), \hat{n}_a + \hat{n}_b] = 0.$$

$$\begin{aligned}\mathrm{BS}(\pi,0)|\Psi_a,\Psi_b\rangle &= e^{-i\frac{\pi}{2}[\hat{n}_a+\hat{n}_b]}\mathrm{SWAP}|\Psi_a,\Psi_b\rangle \\ &= e^{-i\frac{\pi}{2}[\hat{n}_a+\hat{n}_b]}|\Psi_b,\Psi_a\rangle,\end{aligned}$$

Generate entanglement under Fock basis between two modes:

Box IV.5. Two-Mode Squeezing Gate

$$\mathrm{TMS}(r,\varphi) = e^{-i\hat{V}_{\mathrm{TMS}}t} = e^{r\left(e^{i\varphi}\, a^\dagger b^\dagger - e^{-i\varphi}\, ab\right)}, \tag{183}$$

where $r = g_{\mathrm{TMS}}t$. The corresponding symplectic mode transformation matrix $\mathbf{TMS}(r,\varphi)$ *is given in Eq. (182). The two-mode squeezed vacuum state [279]*

$$\mathrm{TMS}(r,\varphi)|0,0\rangle = \frac{1}{\cosh r}\sum_{m=0}^{\infty}\left[e^{i\varphi}\tanh r\right]^m |m,m\rangle, \tag{184}$$

is an entangled state of two modes characterized by having equal photon numbers in both modes. For $\varphi = 0$ one can think of this entangled state as the CV analog of the DV Bell state

$$|\mathrm{Bell}\rangle = \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right], \tag{185}$$

which is characterized by an equal excitation number in both qubits.

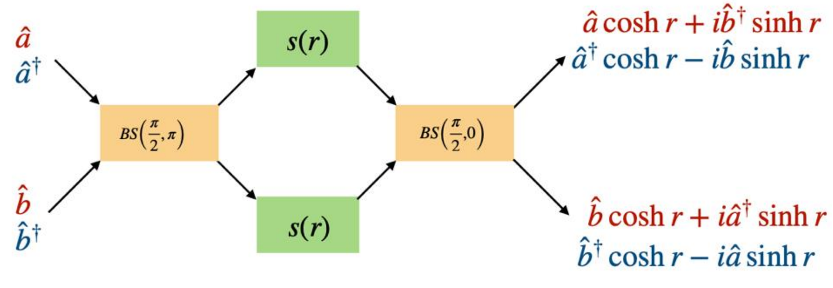
Bloch-Messiah Decomposition

Any 2N ×2N real symplectic matrix M can be decomposed into the form

$$M = \boldsymbol{O_1}\,\mathbf{Z}\,\boldsymbol{O_2}$$

$$\mathbf{Z} = \mathrm{Diag}\left(e^{r_1}, e^{-r_1}, e^{r_2}, e^{-r_2}, \ldots, e^{r_N}, e^{-r_N}\right)$$

- Allows decomposition of arbitrary two-mode gate into number preserving and single-mode squeezing.



Box IV.6. Two-Mode Sum Gate

$$\mathrm{SUM}(\lambda) = e^{-i2\lambda\hat{x}_a\hat{p}_b} = e^{\frac{\lambda}{2}(a+a^\dagger)(b^\dagger - b)} \tag{192}$$

$$\mathrm{SUM}(\lambda)|x_a\rangle|x_b\rangle = |x_a\rangle|x_b + \lambda x_a\rangle \tag{193}$$

where λ is an arbitrary real scale factor. The factor of 2 comes from the choice of Wigner units in which the generator of displacements is $2\hat{p}$ as shown in Eq. (58). In the momentum representation, the action of the SUM gate is

$$\mathrm{SUM}(\lambda)|p_a\rangle|p_b\rangle = |p_a - \lambda p_b\rangle|p_b\rangle \tag{194}$$

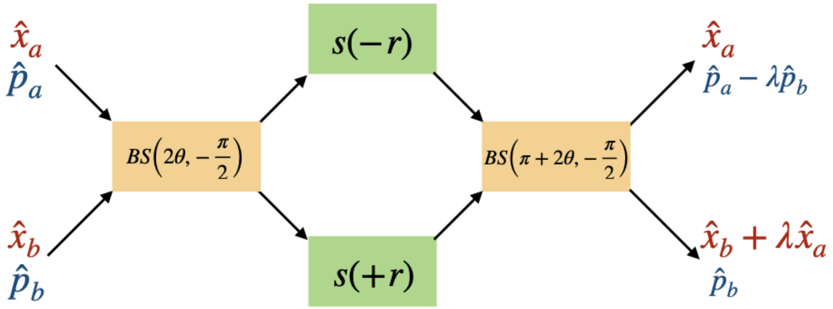
The SUM gate can be used to create Bell-like entangled states of the following form in the position representation

$$\begin{aligned}\mathrm{SUM}(\lambda{=}1)&\int_{-\infty}^{+\infty} \mathrm{d}x\,\Psi(x)\left[|x\rangle \otimes S(r,0)|0\rangle\right] \\ &\approx \int_{-\infty}^{+\infty} \mathrm{d}x\,\Psi(x)\left[|x\rangle \otimes |x\rangle\right],\end{aligned} \tag{195}$$

where we have assumed $r \gg 1$ so that the squeezed vacuum state $S(r,0)|0\rangle$ is approximately a position eigenstate centered at position 0.

This allows quantum arithmetic on continuous-variables!

Generate position correlated (entangled) states.



CV-DV comparison

Aspect	Qubits (DV)	Bosonic Modes (CV)
Standard Basis	Z, $\{0,1\}$	$\hat{x}$, $\lvert x\rangle$
Conjugate Basis	X, $\lvert 0\rangle \pm \lvert 1\rangle$	$\hat{p}$, $\lvert p\rangle = \int dx\, e^{ixp}\lvert x\rangle$
Basis Transformation	Hadamard	Fourier Transform (i.e., $e^{i\frac{\pi}{2}\hat{n}}$)
Group	Pauli	Weyl-Heisenberg (e.g., $e^{i\alpha\hat{x}}$, $e^{i\beta\hat{p}}$)
	Clifford	Gaussian unitaries (e.g., $e^{i\theta\hat{n}}, e^{i\lambda\hat{x}^2}, e^{i\hat{x}_1\hat{p}_2}$)
Channel	Pauli/Clifford	Displacement/Gaussian
Measurement	Pauli basis Measurement of X, Z requires Clifford operations	Homodyne Measurement of $\hat{x}, \hat{p}$ requires Gaussian operations
	Non-Clifford $(\vec{b}\cdot\vec{\sigma})$	Non-Gaussian $(a^\dagger a, \lvert n\rangle\langle n\rvert, aa, \hat{x}^2, \hat{p}^3, \cos k\hat{x})$
Representation	State Tomography	Wigner (or Characteristic) Function Tomography
Benchmarking	Clifford operations are 2-design	Gaussian operations are not 2-design
Example Entangled State	Bell: $\lvert 00\rangle + \lvert 11\rangle$	EPR (TMS, SUM): $\sum_n \lvert n\rangle_1 \lvert n\rangle_2$, $\int dx\, \lvert x\rangle_1 \lvert x\rangle_2$
Universal Computation	a non-Clifford gate, $T: e^{i\pi/8Z}$ augments the Clifford group with magic (non-Cliffordness)	a non-Gaussian unitary, cubic gate: $e^{i\chi\hat{x}^3}$ augments Gaussian operations with Wigner negativity

State Tomography for CV (bosonic) modes

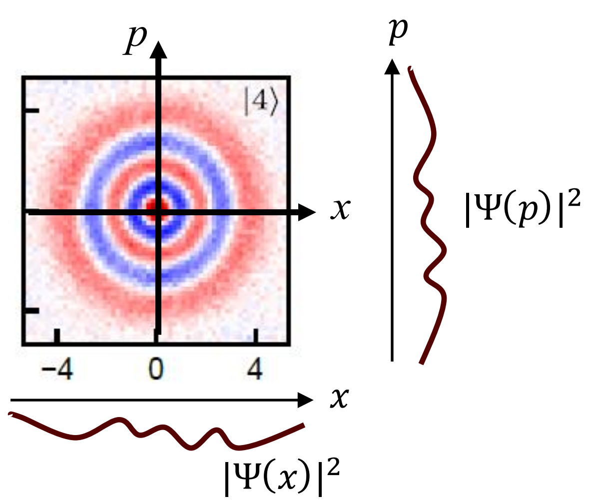
Three equivalent (up to Fourier transforms) representations arranged in order of <u>descending</u> experimental difficulty

Density matrix: $\rho(x,x') = \langle x|\Psi\rangle\langle\Psi|x'\rangle$

Wigner function: $W(\alpha) = \langle\Psi|D^\dagger(-\alpha)\,\hat{P}\,D(-\alpha)|\Psi\rangle; \quad \hat{P} \equiv (-1)^{a^\dagger a}$

Characteristic function: $C(\alpha) = \langle\Psi|D(\alpha)|\Psi\rangle = \langle\Psi|e^{\left[\alpha a^\dagger - \alpha^* a\right]}|\Psi\rangle$

- All three provide complete information about the quantum state.
- Integration of Wigner function over one variable gives probability density (positive).



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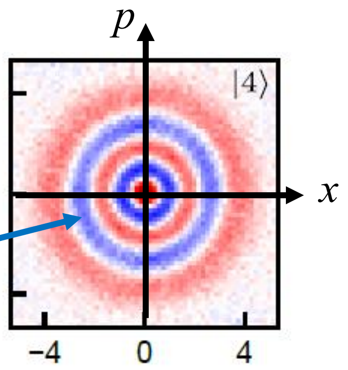


Wigner negativity

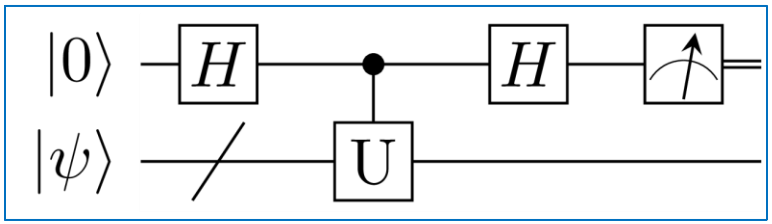
- Wigner negativity causes rapid sign oscillations in the quasi-probability distribution, making it difficult to simulate by classical Monte Carlo to obtain expectation values of observables: $O(\hat{x},\hat{p})$.
- Cannot have true phase space probability distribution because $[\hat{x},\hat{p}] \neq 0$.

Wigner Function Measurement: $W(\alpha) = \langle \Psi | D^{\dagger}(-\alpha)\, \hat{P}\, D(-\alpha) | \Psi \rangle; \quad \hat{P} \equiv (-1)^{a^{\dagger}a} = (-1)^{\hat{n}}$

Measure 'displaced parity' via phase kickback on controlled parity gate

Hadamard Test:

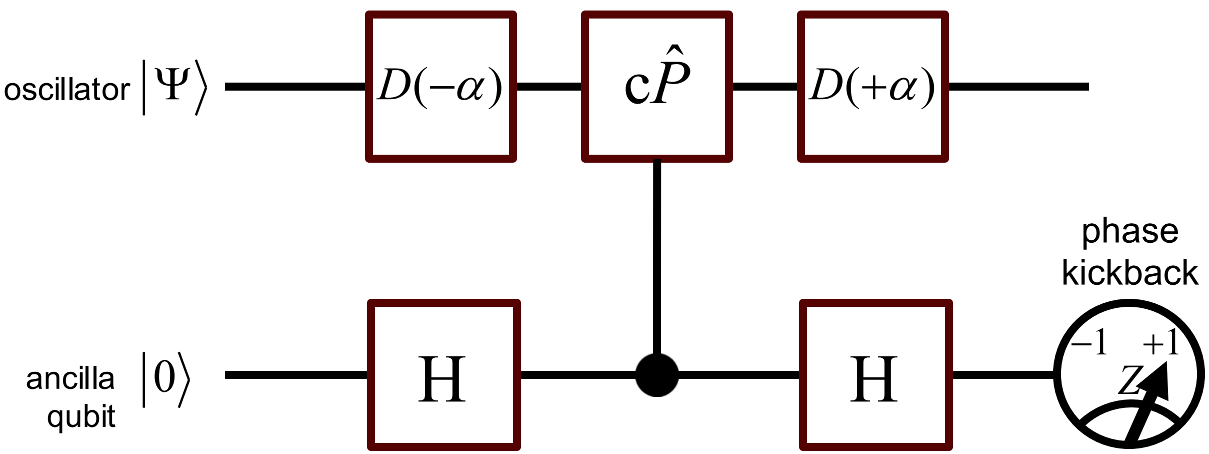
$\mathrm{Re}\langle \psi | U | \psi \rangle$



Controlled parity gate is generated by the dispersive coupling between ancilla and oscillator (*not natively available in ion traps*).

$$H = \chi Z\, \hat{n}$$

$$\mathrm{c}\hat{P} = e^{-i\frac{\pi}{2}\hat{n} Z}$$



Rotates ancilla by angle

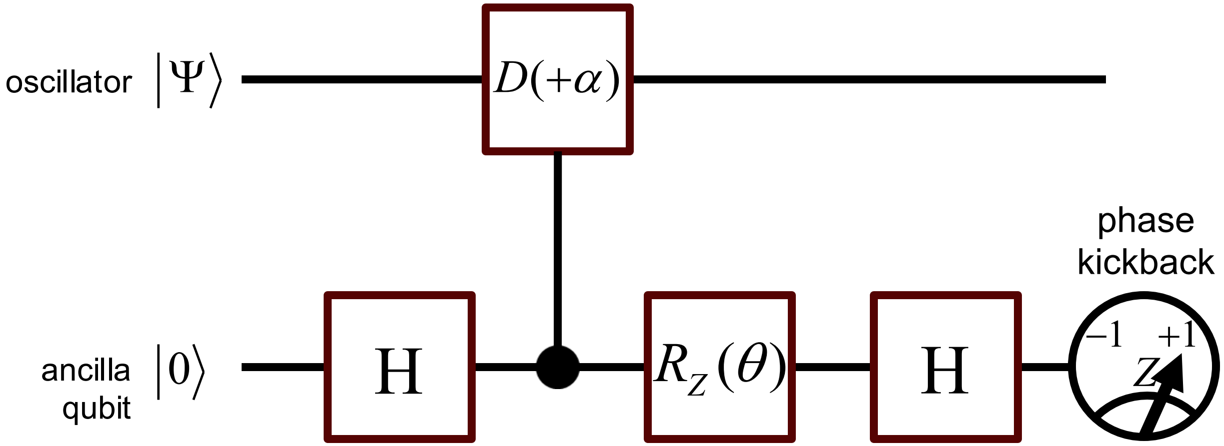
$$\theta = \pi \hat{n}$$

to produce measurement result

$$z = (-1)^{\hat{n}}$$

Characteristic Function Measurement: $C(\alpha) = \langle \Psi | D(\alpha) | \Psi \rangle = \langle \Psi | e^{\left[\alpha a^\dagger - \alpha^* a\right]} | \Psi \rangle$

Measure 'displacement operator' via phase kickback on controlled displacement gate



Controlled displacement gate
(*also natively available in ion traps*)

$$cD(\alpha) = e^{\left[\alpha a^\dagger - \alpha^* a\right] Z}$$

$$\langle Z \rangle = \mathrm{Re}\left[e^{i\theta} C(\alpha)\right]$$

Eigenvalues of the unitary $C(\alpha)$ lie on the unit circle: $e^{i\varphi}$

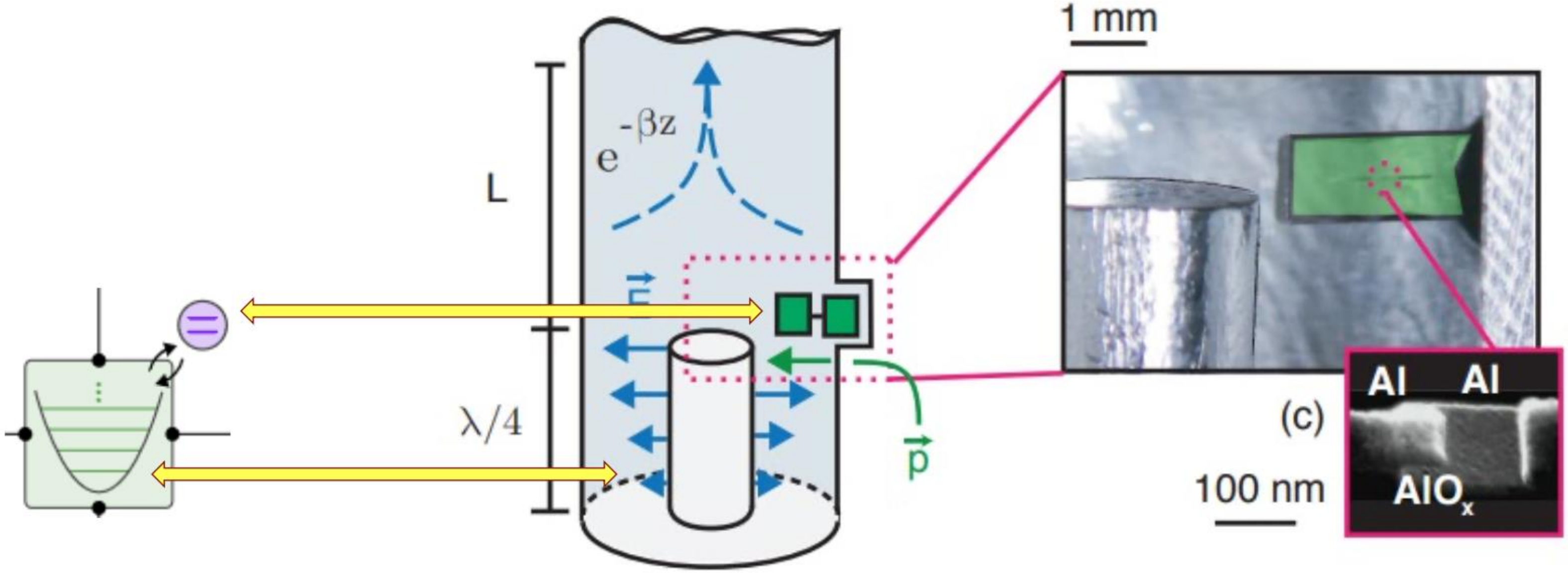
One-bit phase estimation:

$$\langle Z \rangle = \mathrm{Re}\left[C(\alpha)\right]; \quad \theta = 0$$

$$\langle Z \rangle = \mathrm{Im}\left[C(\alpha)\right]; \quad \theta = -\pi/2$$

What about experimental realizations?

Cavity Quantum Electrodynamics with Superconducting Circuits



Coupling strength and frequency is set once fabricated.

Cavity: $T_1 \approx 1\,ms - 30\,ms$
$T_\phi > 100\,ms$
Transmon: $T_1 \approx 100\,\mu s - 300\,\mu s$
$T_\phi \approx 100\,\mu s - 1\,ms$

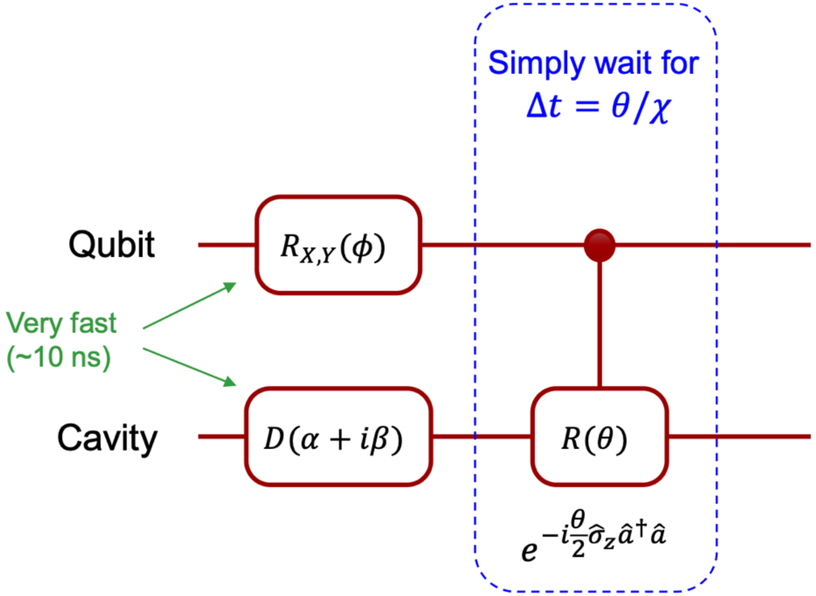
$$\frac{\hat{H}}{\hbar} = \omega_A \hat{a}^\dagger \hat{a} + \omega_q \frac{\hat{\sigma}_z}{2} - \chi_{Aq} \hat{a}^\dagger \hat{a} \frac{\hat{\sigma}_z}{2} + \cdots$$

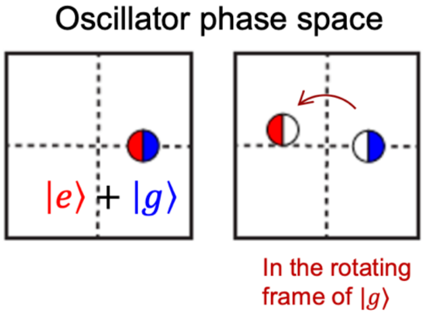
Dispersive interaction (resource)

M. Reagor et al. PRB (2016)

Figure by courtesy of Prof. Chen Wang (UMass Amherst)

Native Qubit-Cavity Operations





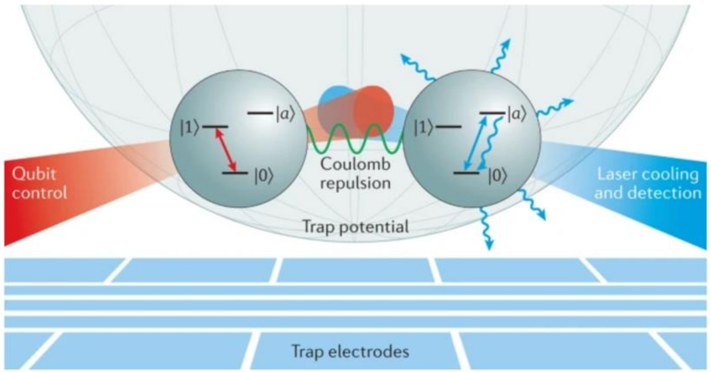
- **The native qubit-controlled cavity phase gate cannot be turned off !**
- **No real idling gate.**

Universal controlling of oscillator with SNAP + Displacement

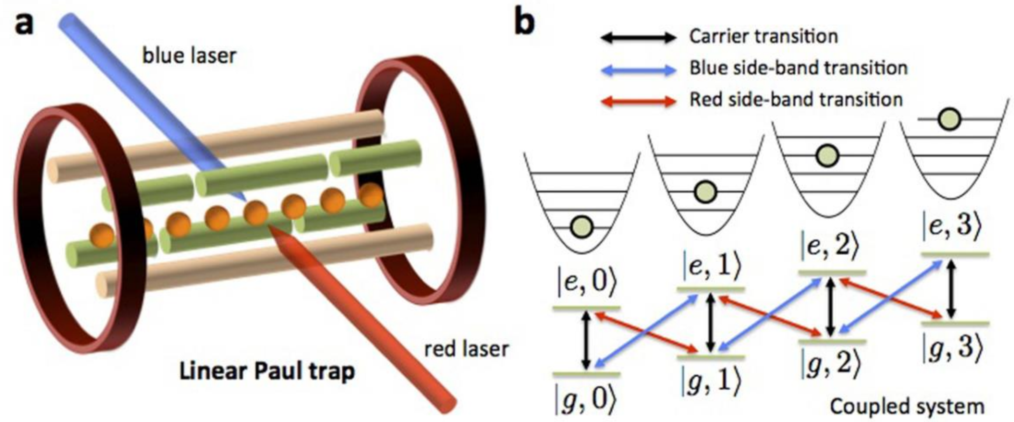
$$SNAP(\vec{\varphi}) = \sum_n e^{-i\varphi_n}|n\rangle\langle n|$$

$$D(\alpha) = \exp[\alpha\hat{a}^+ - \alpha^*\hat{a}]$$

Trapped Ion Quantum Computer



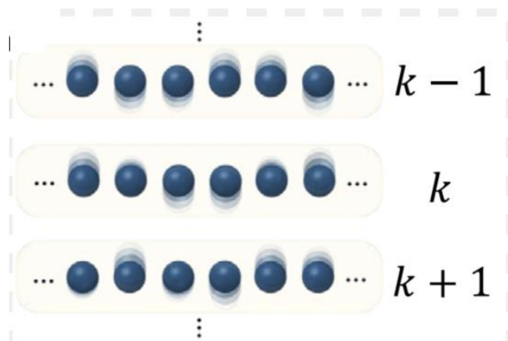
Nature Reviews Materials **6**, 892–905 (2021).



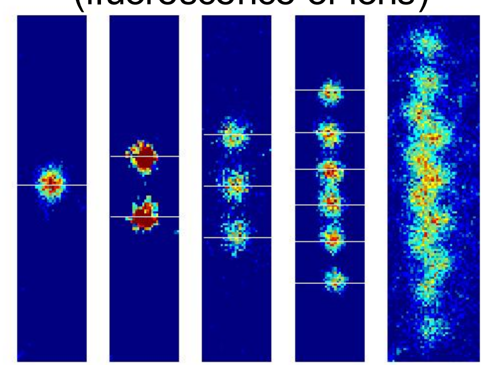
Scientific Reports **4**, 3589 (2014).

Qubit or qudit -> Ion
Oscillators: Ions' collective motion / vibration

- Qubit-oscillator interaction can be switched on-off by laser
- One qubit can couple to many modes



Readout / Measurement (fluorescence of ions)



Jaynes-Cummings Gate

$$\mathrm{JC}(\theta, \varphi) = \exp\left[-i\theta\left(e^{i\varphi}\sigma_- a^\dagger + e^{-i\varphi}\sigma_+ a\right)\right]$$

Oscillator-Only Gates

		Gate Name	Parameters	Definition	Reference Theory/Exp.
Oscillator-only Gates	Gaussian (Sec. IV B)	Displacement (Box IV.1)	$\alpha \in \mathbb{C}$	$D(\alpha) = \exp\left[\alpha a^\dagger - \alpha^* a\right]$	[279]/ [29]
		Phase-space rotation (Box IV.2)	$\theta \in [0, 2\pi)$	$R(\theta) = \exp\left[-i\theta a^\dagger a\right]$	[214]/
		Fourier transform	$\theta = \pi/2$	$F = \exp\left[-i\frac{\pi}{2} a^\dagger a\right]$	
		(Single-mode) squeezing (Box IV.3)	$\zeta \in \mathbb{C}$	$S(\zeta) = \exp\left[\frac{1}{2}(\zeta^* a^2 - \zeta a^{\dagger 2})\right]$	[279]/ [274]
		Beam-splitter (Box IV.4)	$\theta \in [0, 4\pi), \varphi \in [0, \pi)$	$\mathrm{BS}(\theta, \varphi) = \exp\left[-i\frac{\theta}{2}\left(e^{i\varphi} a^\dagger b + e^{-i\varphi} a b^\dagger\right)\right]$	[279, 294]/ [112, 229]
		SWAP	$\theta = \pi, \varphi = \pi/2$	$\mathrm{SWAP} = \exp\left[\frac{\pi}{2}(a^\dagger b - a b^\dagger)\right]$	
		Two-mode squeezing (Box IV.5)	$r \in \mathbb{R}, \varphi \in [0, \pi)$	$\mathrm{TMS}(r, \varphi) = \exp\left[r\left(e^{i\varphi} a^\dagger b^\dagger - e^{-i\varphi} a b\right)\right]$	[279]/ [284]
		Two-mode summing (Box IV.6)	$\lambda \in \mathbb{R}$	$\mathrm{SUM}(\lambda) = \exp\left[\frac{1}{2}\lambda\left(a + a^\dagger\right)\left(b^\dagger - b\right)\right]$	[211]/ [280, 295]
	Non-Gaussian (Sec. IV D)	Generalized single-mode squeezing[1]	$z \in \mathbb{C}, N \geq 3$	$U_N(z) = \exp\left[z a^{\dagger N} - z^* a^N\right]$	[296]/ [297, 298]
		Generalized k-mode squeezing	$z \in \mathbb{C}, k \geq 3$ $\vec{n} = \{n_1, \cdots, n_k\}$	$U_{\vec{n}}(z) = \exp\left[z \prod_{p=1}^{k} a_p^{\dagger n_p} - z^* \prod_{p=1}^{k} a_p^{n_p}\right]$	[299]/ [297, 300]
		Cubic-phase	$r \in \mathbb{R}$	$C(r) = \exp\left[-ir\hat{x}^3\right]$	[301, 302]/ [254, 297, 298]
		Self-Kerr	$\theta \in \mathbb{R}$	$K(\theta) = \exp\left[-i\frac{\theta}{2} a^{\dagger 2} a^2\right]$	[209, 279] / [303]
		SNAP[2] (Box IV.10)	$\vec{\varphi} = \{\varphi_n\}, \varphi_n \in [0, 2\pi)$	$\mathrm{SNAP}(\vec{\varphi}) = \sum_n e^{-i\varphi_n} \lvert n\rangle\langle n\rvert$	[55]/[253]
		eSWAP (Box IV.11)	$\theta \in [0, 4\pi)$	$\exp\left[-i\frac{\theta}{2}\left(\mathrm{SWAP}\right)\right]$	[304]/[305]

Hybrid Oscillator-Qubit Gates

		Gate Name	Parameters	Definition
Hybrid Oscillator-Qubit Gates	Single-Oscillator (Sec. IV D)	Conditional rotation (Box IV.8)	$\theta \in [0, 4\pi)$	$\mathrm{CR}(\theta) = \exp\left[-i\frac{\theta}{2}\sigma_z a^\dagger a\right]$
		Conditional parity (Box IV.8)	$\theta = \pi$	$\mathrm{CP} = \exp\left[-i\frac{\pi}{2}\sigma_z a^\dagger a\right]$
		SQR[1] [311] (Box IV.9)	$\theta_n \in [0, 4\pi)$ $\varphi_n \in [0, 2\pi)$	$\mathrm{SQR}(\vec{\theta}, \vec{\varphi}) = \sum_n R_{\varphi_n}(\theta_n) \otimes \|n\rangle \langle n\|$
		Jaynes-Cummings[2]	$\theta, \varphi \in [0, 2\pi)$	$\mathrm{JC}(\theta, \varphi) = \exp\left[-i\theta\left(e^{i\varphi}\sigma_- a^\dagger + e^{-i\varphi}\sigma_+ a\right)\right]$
		Anti-Jaynes-Cummings[3]	$\theta, \varphi \in [0, 2\pi)$	$\mathrm{AJC}(\theta, \varphi) = \exp\left[-i\theta\left(e^{i\varphi}\sigma_+ a^\dagger + e^{-i\varphi}\sigma_- a\right)\right]$
		Conditional displacement [56, 60] (Box IV.7)	$\alpha \in \mathbb{C}$	$\mathrm{CD}(\alpha) = \exp\left[\sigma_z\left(\alpha a^\dagger - \alpha^* a\right)\right]$
		Rabi interaction	$\theta \in \mathbb{R}$	$\mathrm{RB}(\theta) = \exp\left[-i\sigma_x(\theta a^\dagger + \theta^* a)\right]$
		Conditional squeezing	$\zeta \in \mathbb{C}$	$\mathrm{CS}\left(\zeta\right) = \exp\left[\frac{1}{2}\sigma_z(\zeta^* a^2 - \zeta a^{\dagger 2})\right]$
	Multi-Oscillator (Sec. IV D)	Conditional beam-splitter [229]	$\theta \in [0, 4\pi)$ $\varphi \in [0, \pi)$	$\mathrm{CBS}(\theta, \varphi) = \exp\left[-i\frac{\theta}{2}\sigma_z(e^{i\varphi}a^\dagger b + e^{-i\varphi}ab^\dagger)\right]$
		Conditional two-mode squeezing	$\xi \in \mathbb{C}$	$\mathrm{CTMS}(\xi) = \exp\left[\sigma_z(\xi a^\dagger b^\dagger - \xi^* ab)\right]$
		Conditional SUM gate	$\lambda \in \mathbb{R}$	$\mathrm{CSUM}(\lambda) = \exp\left[\frac{\lambda}{2}\sigma_z(a^\dagger + a)(b^\dagger - b)\right]$

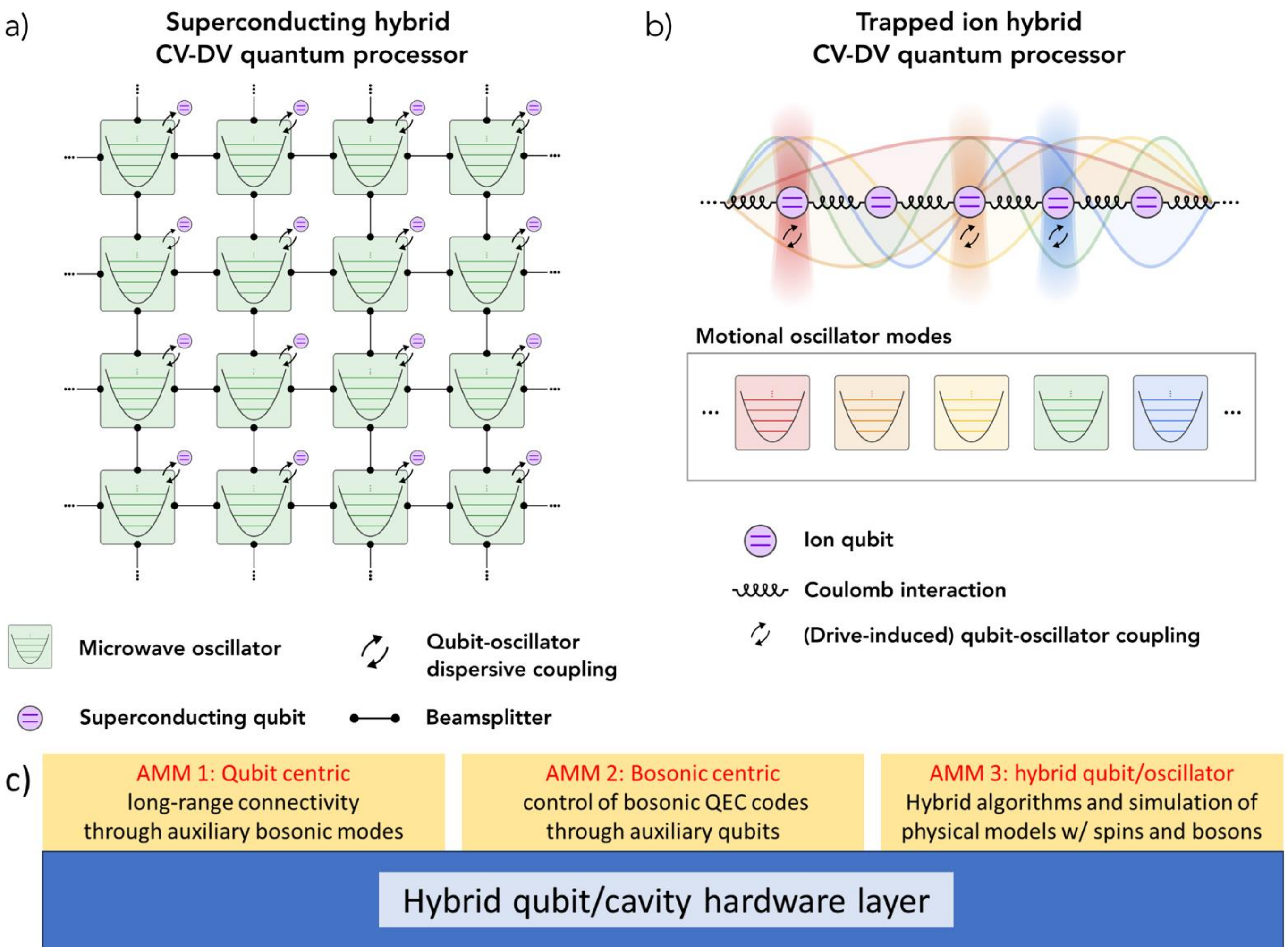
PART II - Instruction Set Architectures, Abstract Machine Models, and Compilation Techniques

- **AMMs:** How to remove some physical-level details to construct abstract machine models for hybrid CV-DV quantum processors?

- **ISAs:** What is a minimal set of gates that are universal?

- **Synthesis and compilation methods:** how to compose the gates together to achieve a desired calculation?

- **Bosonic QEC**



arXiv: 2407.10381

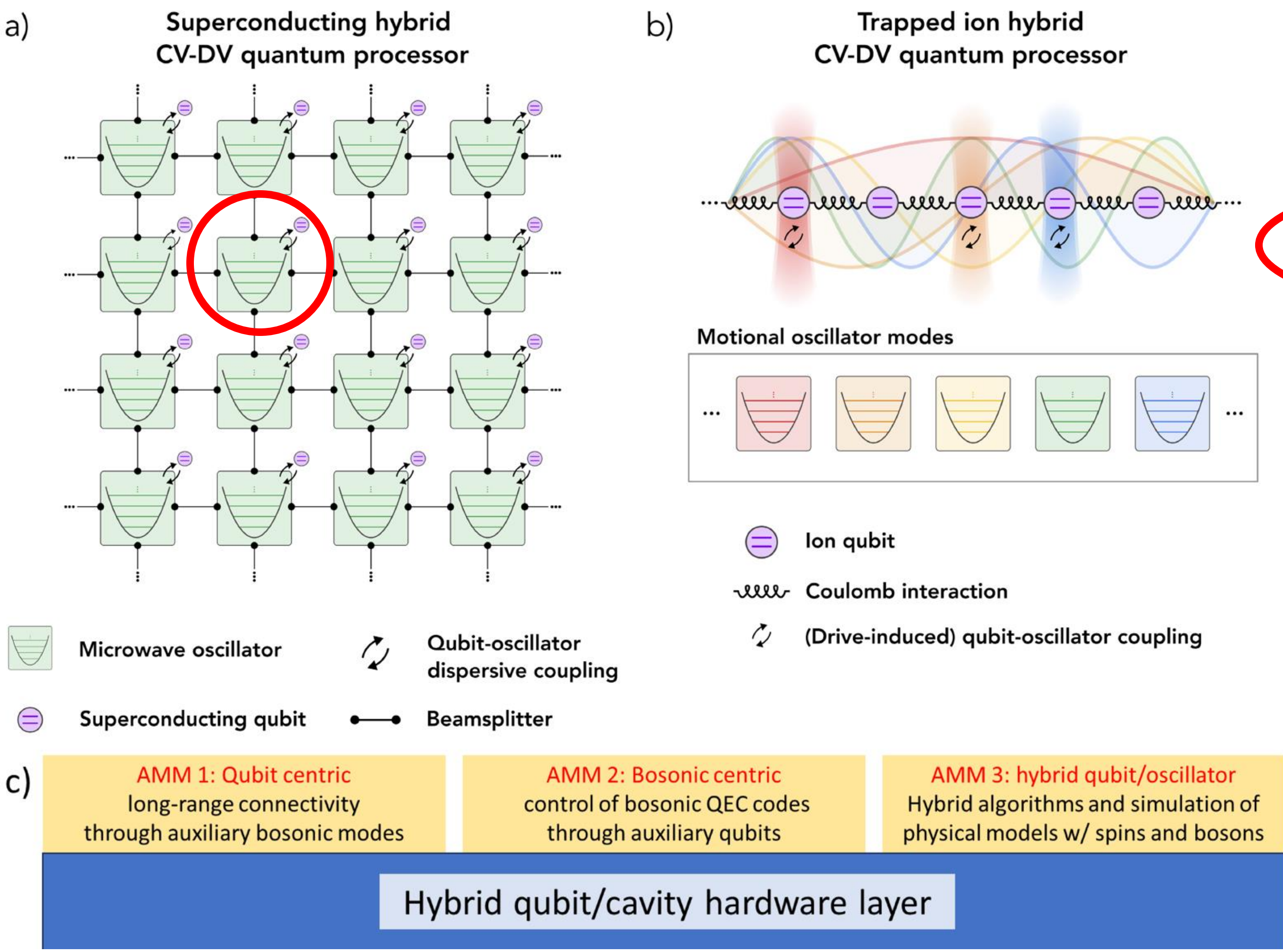
[3D resonator + 1 transmon] local universal control

Minimal cross-talk

High-fidelity parallelizable bosonic communication SWAP network

c.f. ion-trap ‘all-to-all’

3 distinct AMMs (abstract machine models)



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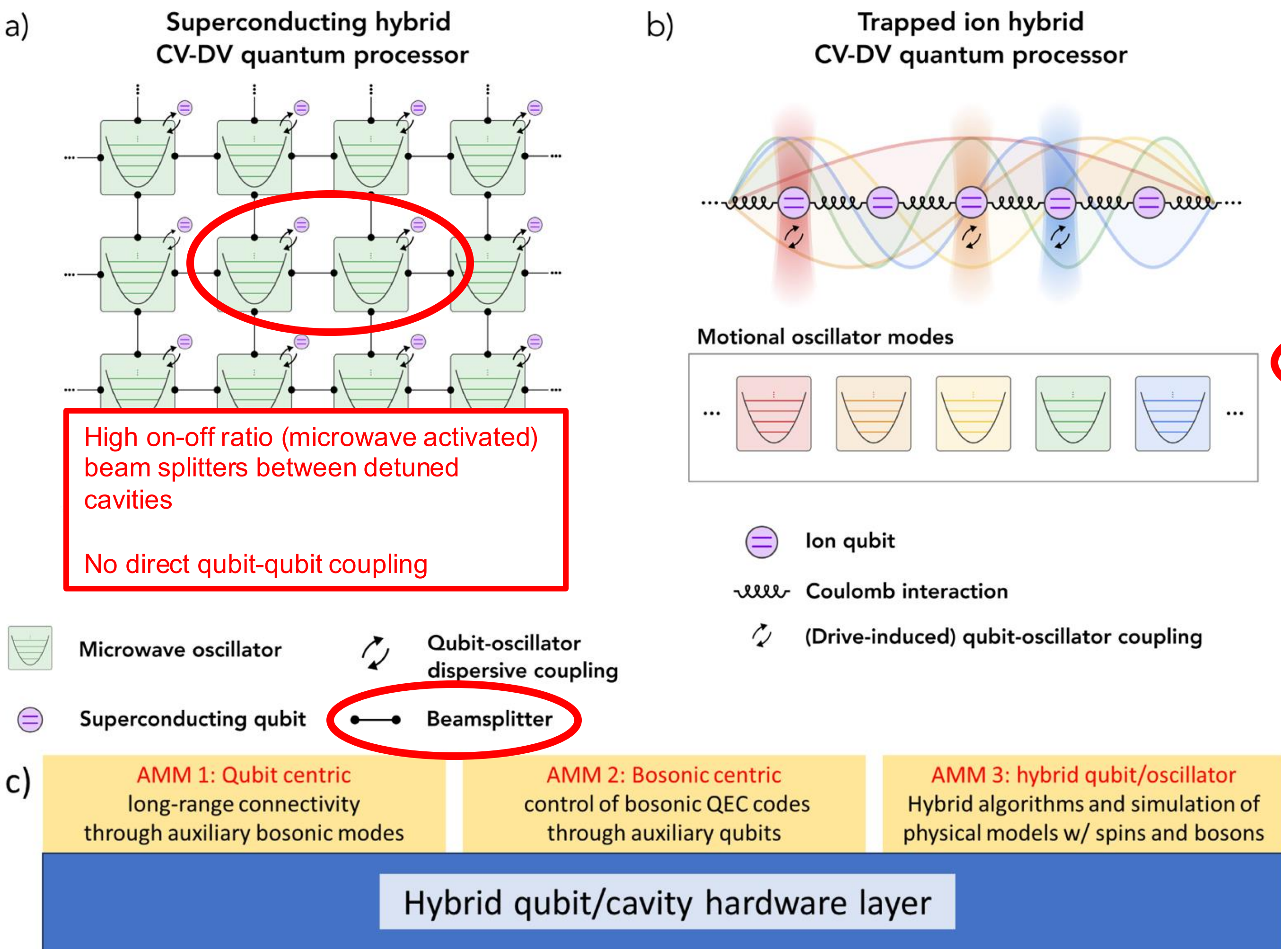

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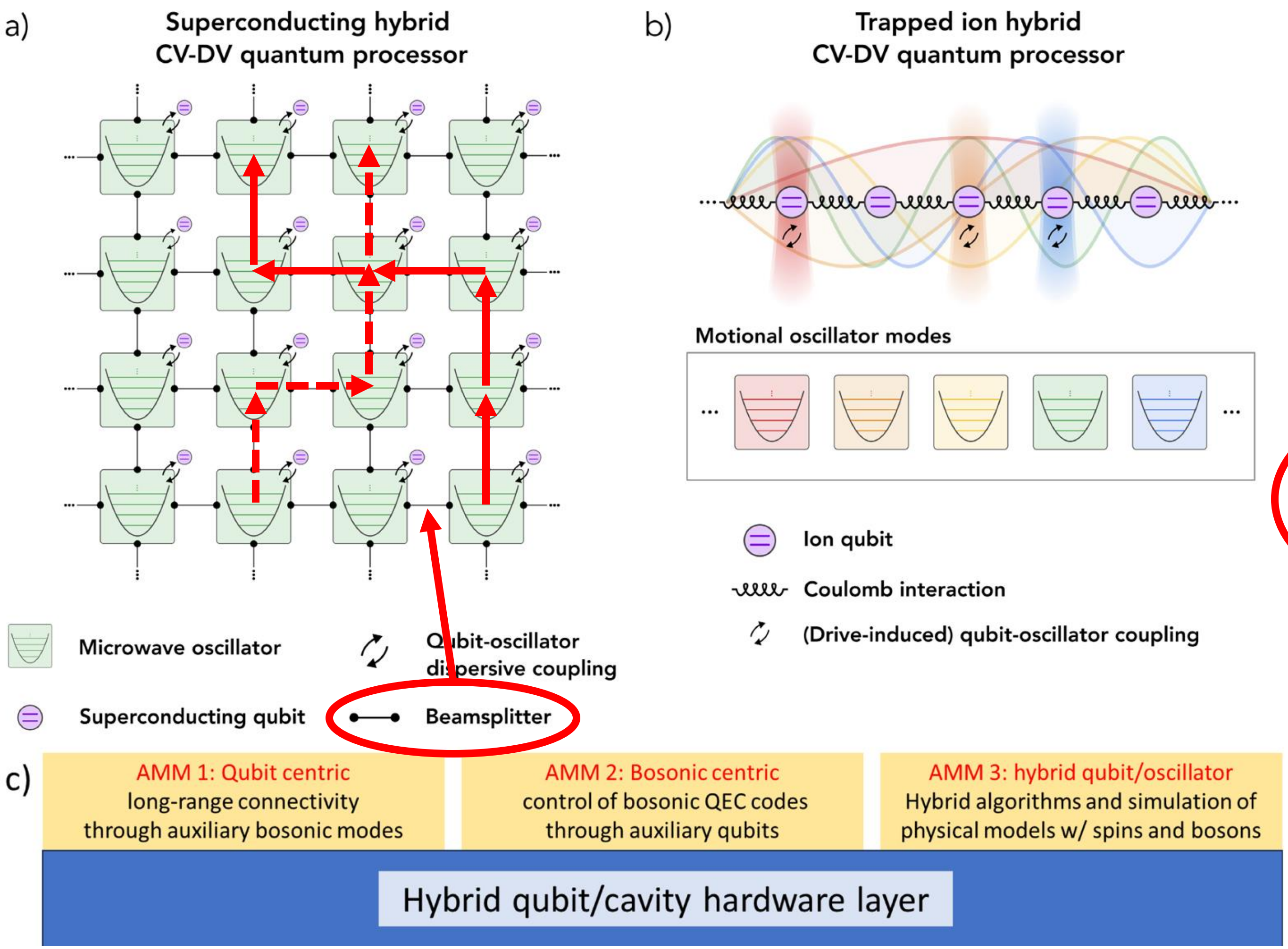

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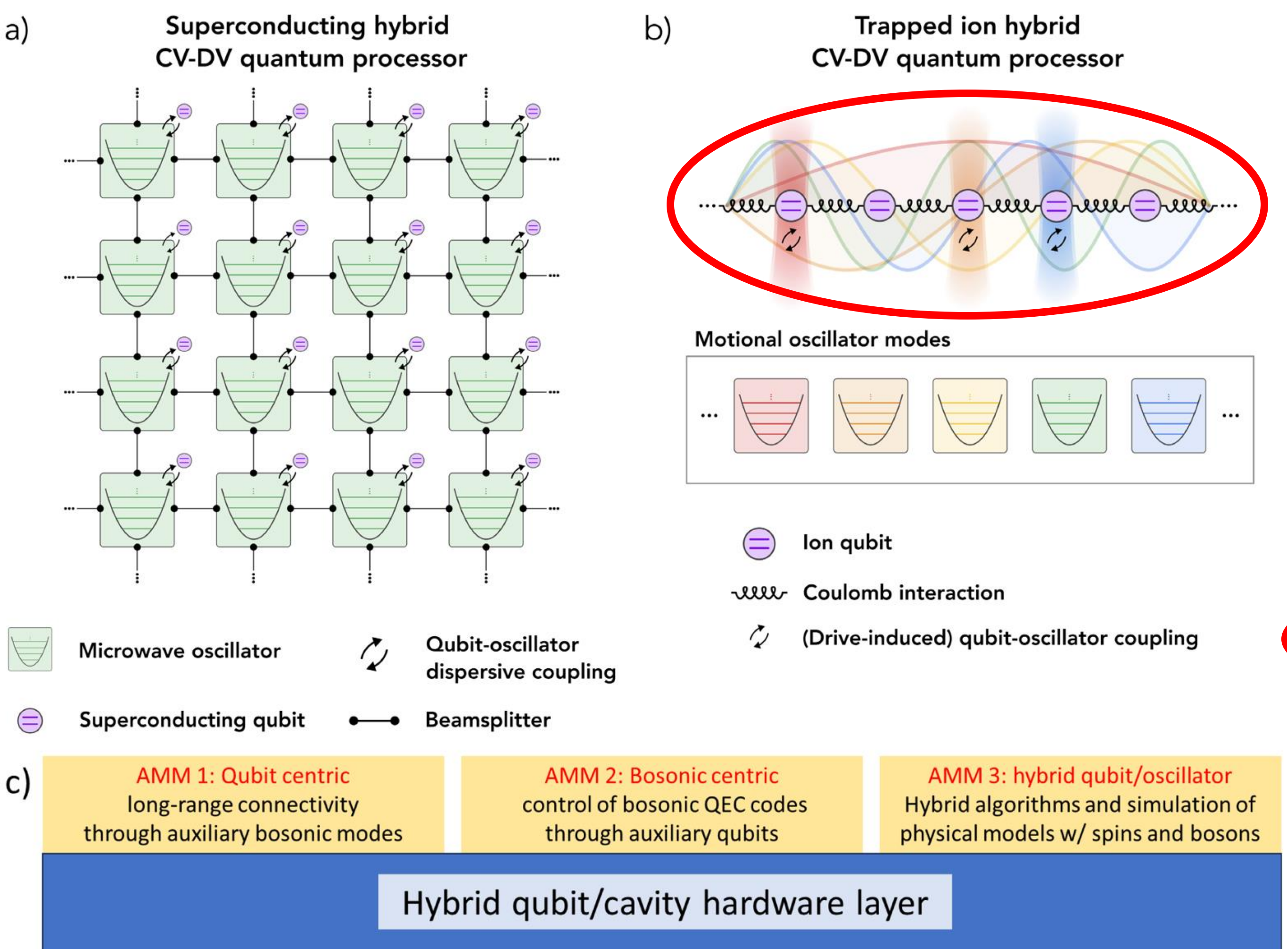

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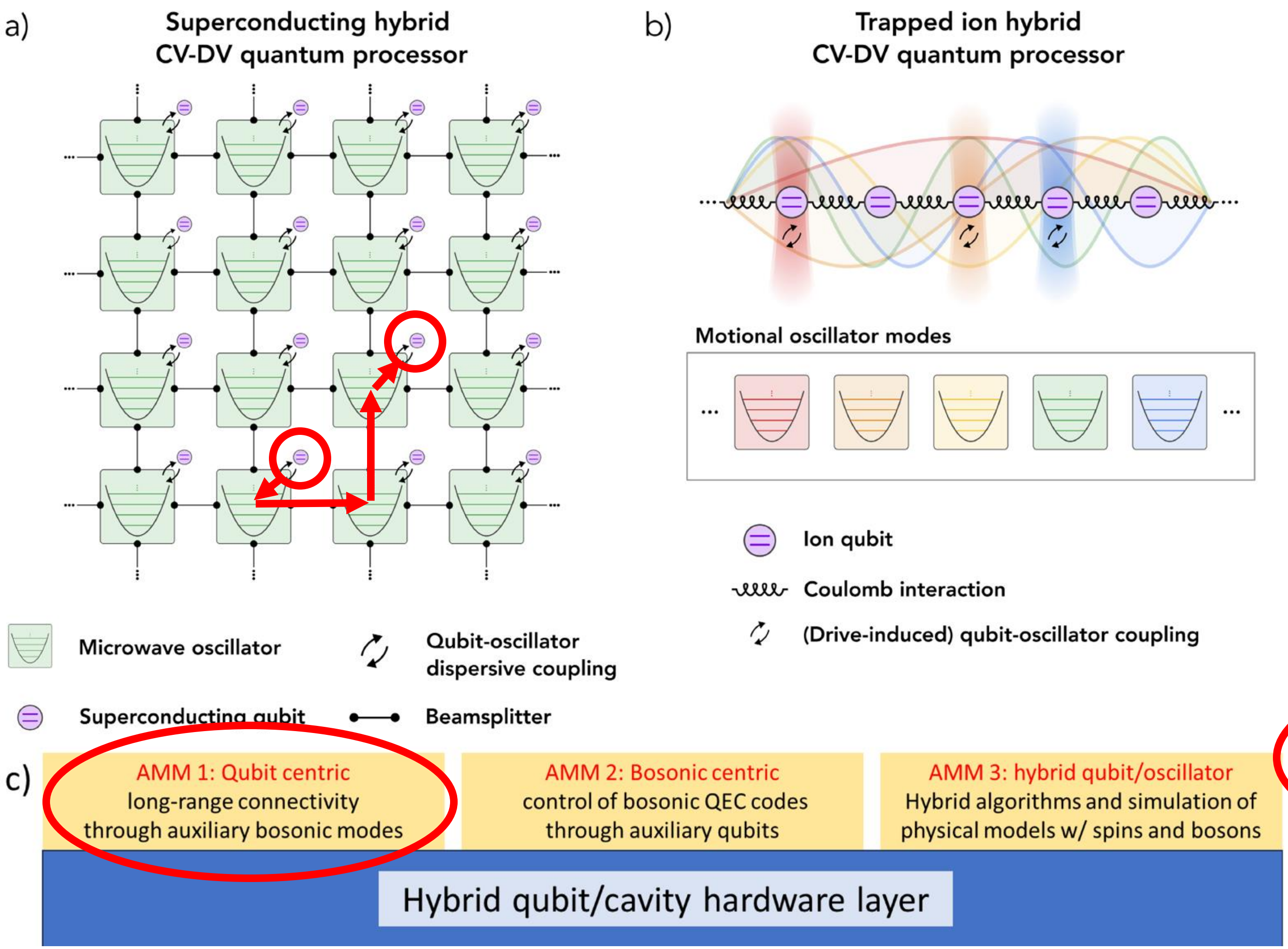

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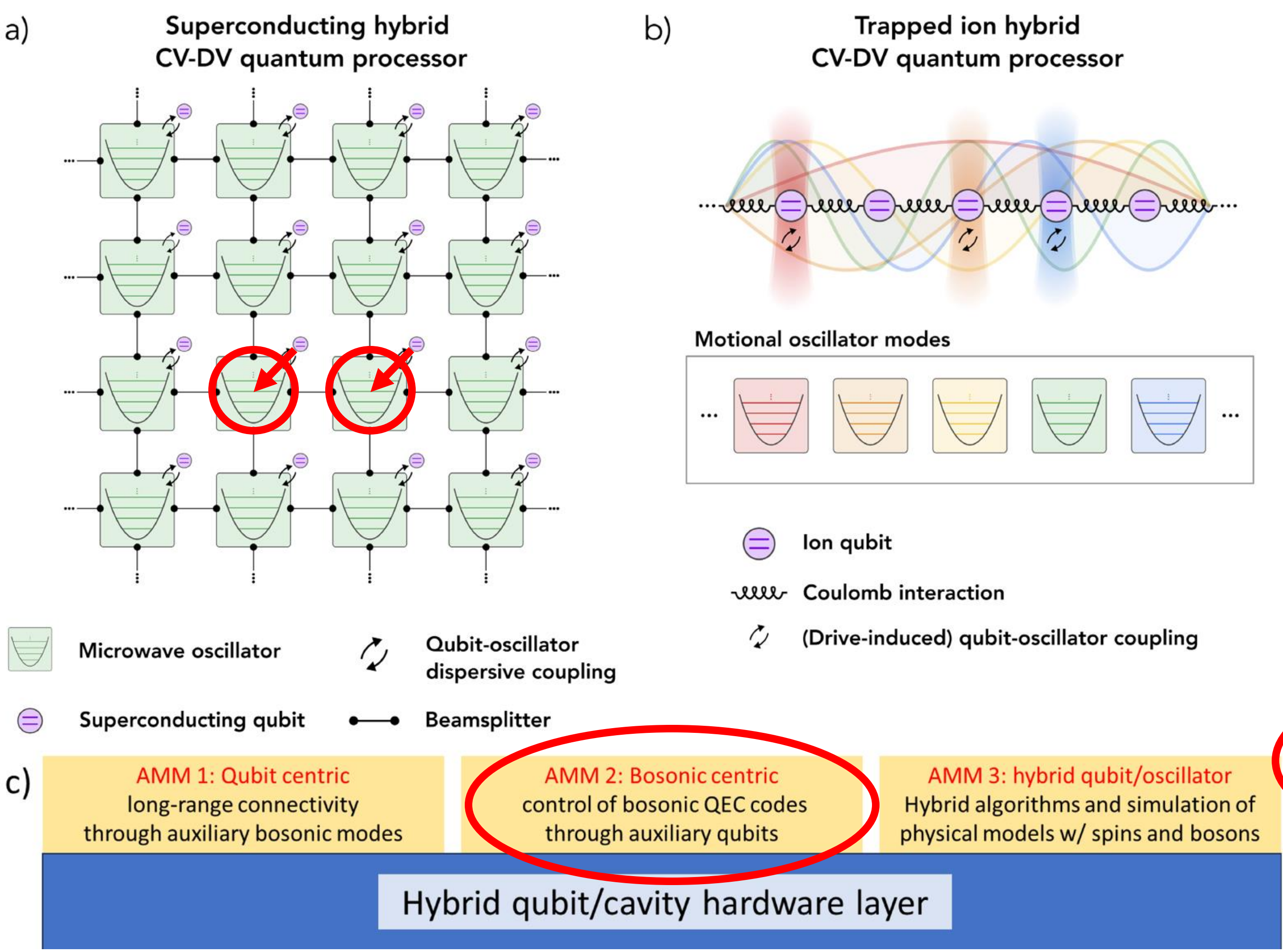

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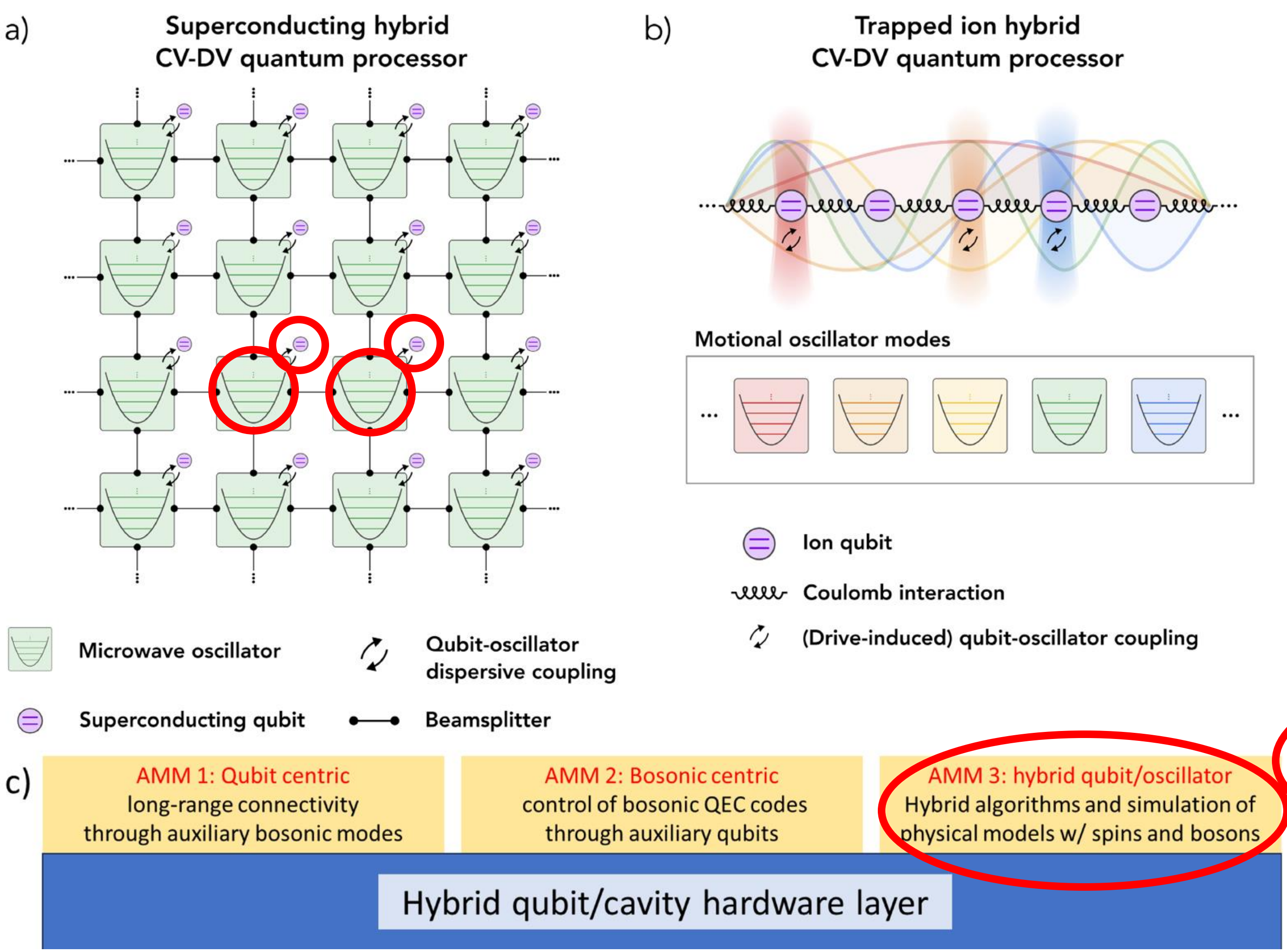

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3 distinct AMMs
(abstract machine models)

Three AMMs and Their Characteristics

AMM Type	Subtype	Physical Qubit Role	Physical Oscillator Role	Physical Qubit Instructions	Physical Bosonic Instructions
QCMM (qubit-centric)	Qubits + Bosonic Memory	Computation	Memory (error corrected)	Universal	Single-oscillator error correction
	Bosonic Bus	Computation	Communication	Universal	Entanglement swapping
	Bosonic Sensing	Computation and I/O	Sensing	Universal	Displacement, Jaynes-Cummings
OCMM (oscillator-centric)	Boson Sampling	I/O	Computation	Boson State preparation and measurement	Displacement, Multi-mode Squeezing, Multi-mode Beam-splitter
	Bosonic System Simulation	I/O	Simulation	Boson State preparation, evolution, and measurement	Universal
HMM (hybrid CV-DV)	Hybrid	Computation Simulation of Spins/Fermions	Computation Simulation	Universal	Universal

$$H = \chi Z\, a^{\dagger} a = \chi Z\, \hat{n}$$

Trapped ions and Circuit QED

'spin dependent forces on oscillator'

$$\hat{x}, \hat{p} \quad \mathrm{CD}(\alpha) = e^{Z(\alpha a^{\dagger} - \alpha^{*} a)}$$

Phase-space instruction set

$\hat{x}, \hat{p}$

- Conditional displacement
- Single qubit rotations
- Beamsplitters

Eickbusch et al., Nat. Phys. (2022)

Circuit QED

'photon number dependent qubit rotations'

$$\hat{n} \qquad \mathrm{SQR} = \exp\left[-i \sum_m \theta_m \sigma^x |m\rangle\langle m| \right]$$

Fock-space instruction set

$|n\rangle$

- Photon number-selective qubit rotations
- Unconditional displacements
- Beamsplitters

Heeres et al., PRL (2015)

Short course on quantum control theory

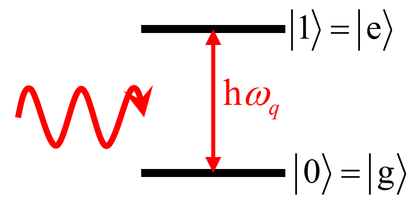
A two-level system (qubit) is controllable with classical drives

$$V(t) = \left[\grave{o}_x(t)\cos(\omega_q t) + \grave{o}_y(t)\sin(\omega_q t) \right] \sigma^x$$

Interaction picture ($H_0 \to 0$) rotating wave approximation:

$$\mathrm{V(t)} \approx \grave{o}_x(t) X + \grave{o}_y(t) Y$$

where X and Y are Pauli matrices in the rotating frame



$$\boxed{H_0 = -\frac{\mathrm{h}\omega_q}{2}\sigma^z}$$

$[X, Y] = 2iZ$

Hence a qubit with two classical controls $\grave{o}_x(t), \grave{o}_y(t)$ is fully controllable.

[Lie Algebra generates the full SU(2) Lie group.]

A quantum system is controllable iff the Lie algebra generated by the available controls spans the space of operators on the Hilbert space, i.e., is full-rank.

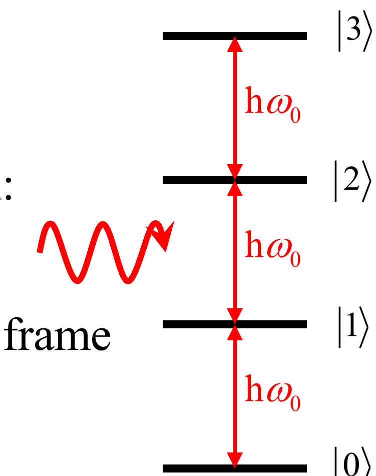
A quantum harmonic oscillator is <u>not</u> controllable with classical drives

$$V(t) = \left[\epsilon_x(t)\cos(\omega_0 t) + \epsilon_y(t)\sin(\omega_0 t)\right](a + a^\dagger)$$

Interaction picture ($H_0 \to 0$) rotating wave approximation:

$$\mathrm{V(t)} \sim \epsilon_x(t)\hat{x} + \epsilon_y(t)\hat{p}$$

where $\hat{x}$ and $\hat{p}$ are phase space coordinates in the rotating frame



$$\boxed{H_0 = \hbar\omega_0\, a^\dagger a}$$

$$[\hat{x}, \hat{p}] = i\hbar$$

Operator algebra closes.

Classical drives can displace/boost the oscillator (e.g.,$\hat{x} \to \hat{x} + \Delta_{\text{classical}}$)
to create coherent states, but cannot create, e.g., Fock states.

In order to fully control a harmonic oscillator, we require
an anharmonic object (e.g., a qubit) as an auxiliary controller.

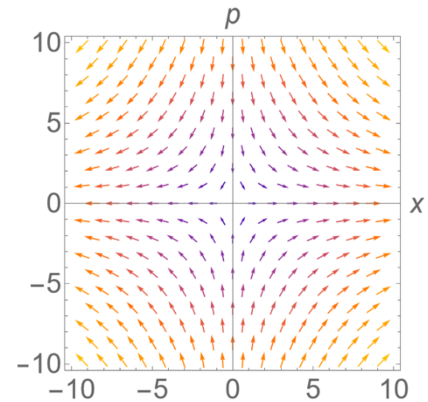
Phase-space Instruction Set (trapped ions and cQED)

$H = \chi\sigma^z a^\dagger a$ [Dispersive]

With drives on the cavity and qubit we can generate ion-trap-like 'spin-dependent' forces. In RWA:

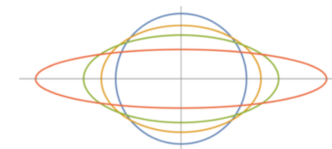
$V = \left[-\epsilon_x(t)\hat{x} + \epsilon_y(t)\hat{p}\right]\left[Z\right] \leftarrow$ can also be X or Y

This in turn generates controlled displacements.



Non-trivial commutator of different available control terms:

$[\hat{x}X, \hat{p}Y] = i(\hat{x}\hat{p} + \hat{p}\hat{x})Z$ (conditional squeezing!)



$[\hat{x}X, \hat{p}Y] = i(\hat{x}\hat{p} + \hat{p}\hat{x})Z$ (conditional squeezing!)

$[\hat{x}X, [\hat{x}X, \hat{p}Y]] = (\hat{x}^2\hat{p} + 2\hat{x}\hat{p}\hat{x} + \hat{p}\hat{x}^2)Y$

Operator algebra does not close $\Rightarrow$ universal control

With Trotter-Suzuki + Baker-Campbell-Hausdorf we can exponentiate sums of operators and products (i.e., commutators) of operators generate universal (effective) Hamiltonian generators of unitaries:

$\mathrm{H} = \vec{\Lambda}(\hat{x}, \hat{p}) \cdot \vec{\sigma} + h(\hat{x}, \hat{p})$ with $\vec{\sigma} = (X, Y, Z)$

Arbitrary polynomials of bounded order

Instruction Sets for Oscillator and Hybrid Oscillator-Qubit Universal Control

	Instruction Set Name	Minimum gate set	Sections
Linear oscillator control	Gaussian	$\mathcal{G} = \{D(\alpha), S(\zeta), \mathrm{BS}(\theta,\varphi) \text{ or } \mathrm{TMS}(r,\varphi)\}$	Sec. IV B and Table IV.2
Universal oscillator control	Cubic	$\mathcal{G} + U_3(z)$	
	Quartic	$\mathcal{G} + U_4(z)$	
	SNAP	$\{D(\alpha), \mathrm{SNAP}(\vec{\varphi}), \mathrm{BS}(\theta,\varphi) \text{ or } \mathrm{TMS}(r,\varphi)\}$	
Universal hybrid control	Phase-Space	$\{\mathrm{CD}(\beta), R_\varphi(\theta), \mathrm{BS}(\theta,\varphi)\}$	Sec. IV D and Table IV.3
	Fock-Space	$\left\{\mathrm{SQR}(\vec{\theta},\vec{\varphi}), D(\alpha), \mathrm{BS}(\theta,\varphi)\right\}$	
	Sideband	$\{R_\varphi(\theta), \mathrm{JC}(\theta), \mathrm{BS}(\theta,\varphi)\}$	

ISA Example: (Echoed) Controlled-Displacement + Qubit Rotations

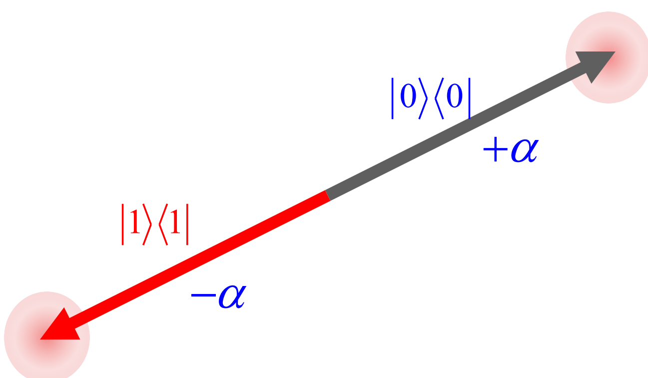
Conditional Displacement Gate

$$D = \exp\left[-it\left(-\grave{o}_x \hat{x} + \grave{o}_y \hat{p}\right)\right] Z$$

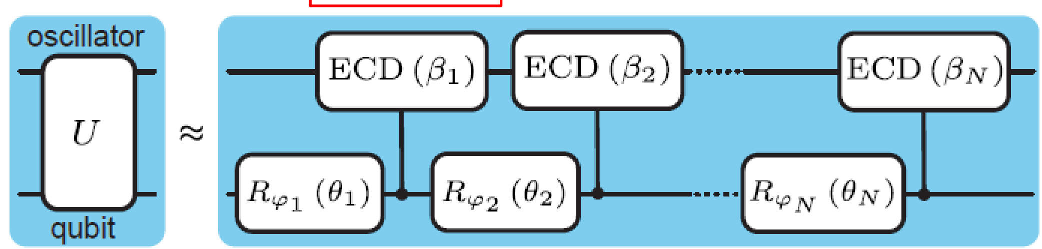
Fast Universal Control of an Oscillator with Weak Dispersive Coupling to a Qubit, A. Eickbusch et al. (Devoret Lab) Nature Physics (2022)

Qubit Rotation Gate

$$R_\varphi(\theta) = e^{-i\frac{\theta}{2}\left[\cos\varphi\,\sigma^x + \sin\varphi\,\sigma^y\right]}$$

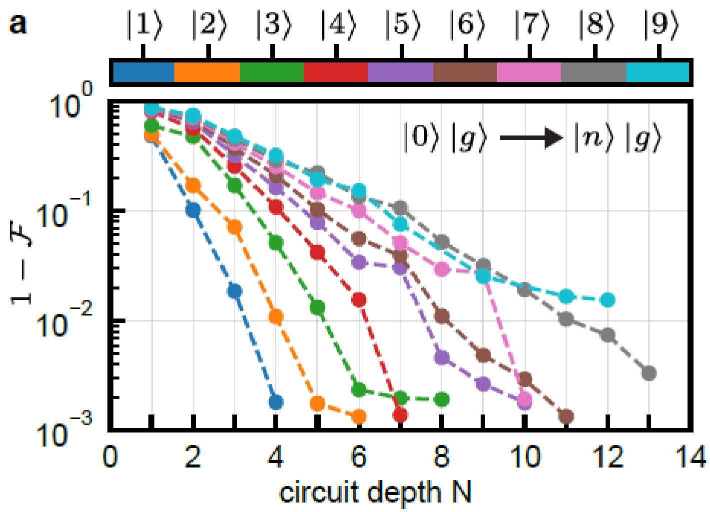


$\mathrm{ECD} = \sigma^x \mathrm{D_c}$



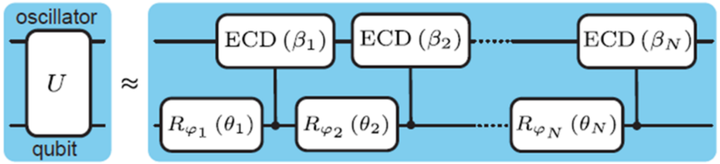
Circuit depth →

(Echoed) Controlled-Displacement ISA:



Eickbusch et al. (Devoret Lab) *Nature Physics* (2022)

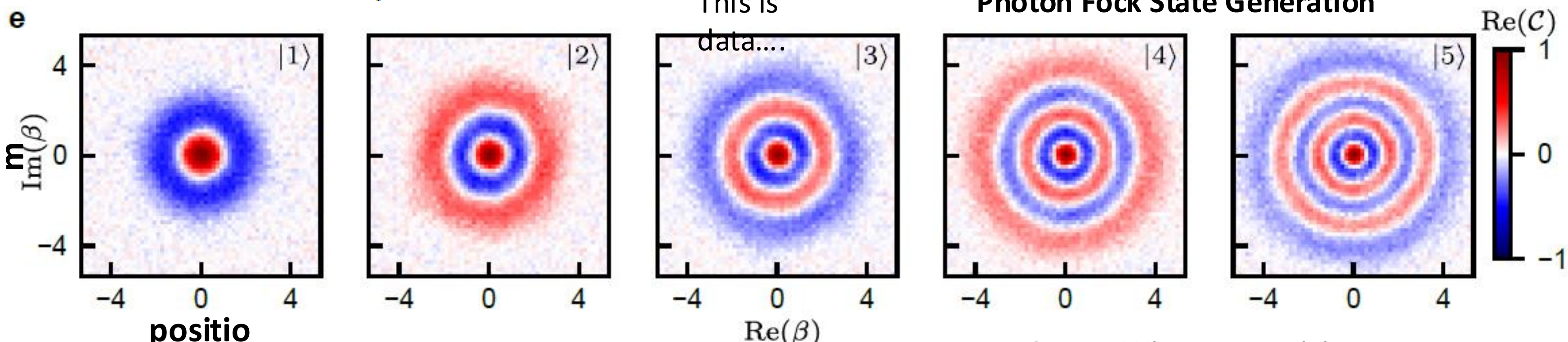
Numerically optimized circuits to solve the disentangling problem and create correct unitary.



Phase Space 'Portraits' of Photon Fock State Generation

This is data….

momentum



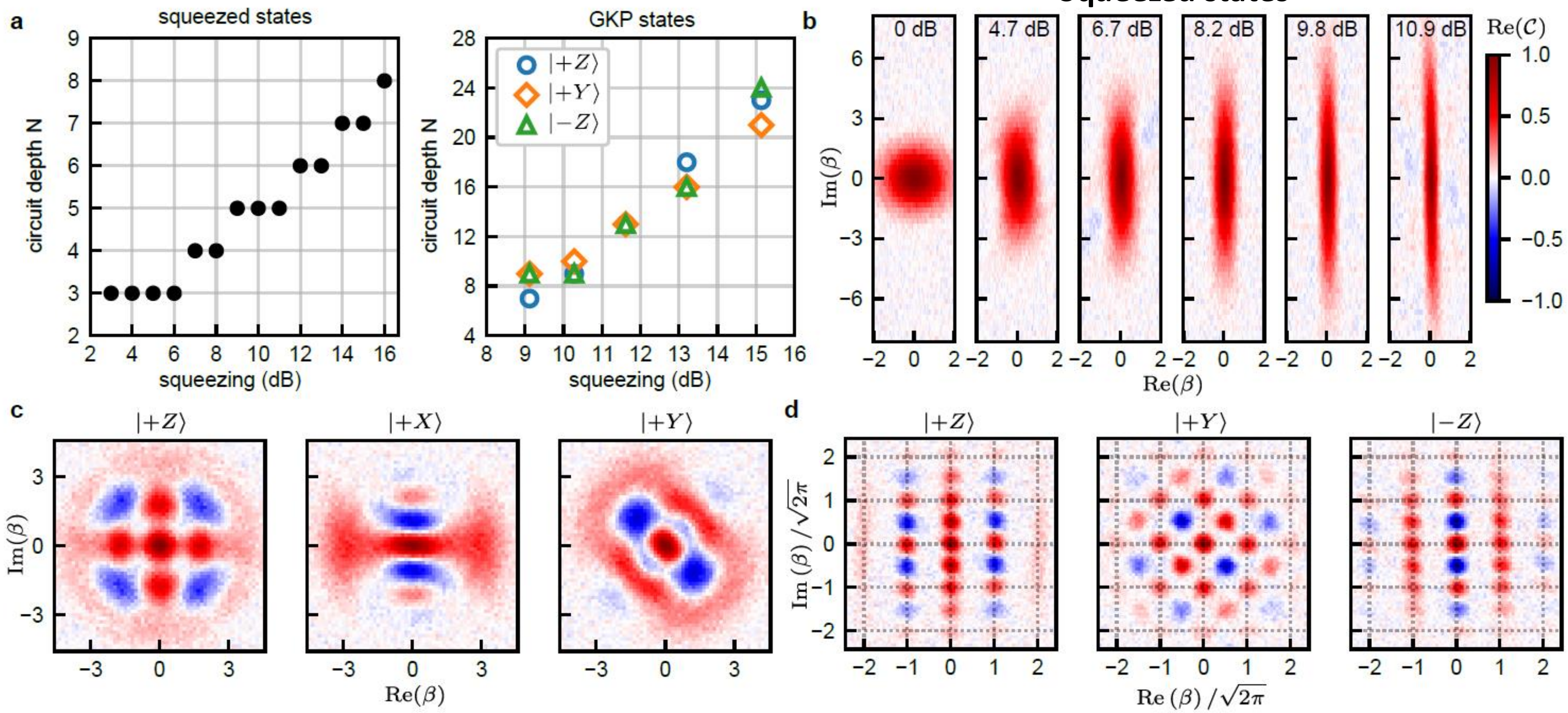
position

Note Strong Wigner negativity

(Echoed) Controlled-Displacement ISA:

Fast Universal Control of an Oscillator with Weak Dispersive Coupling to a Qubit, A. Eickbusch et al. (Devoret Lab) *Nature Physics* (2022)

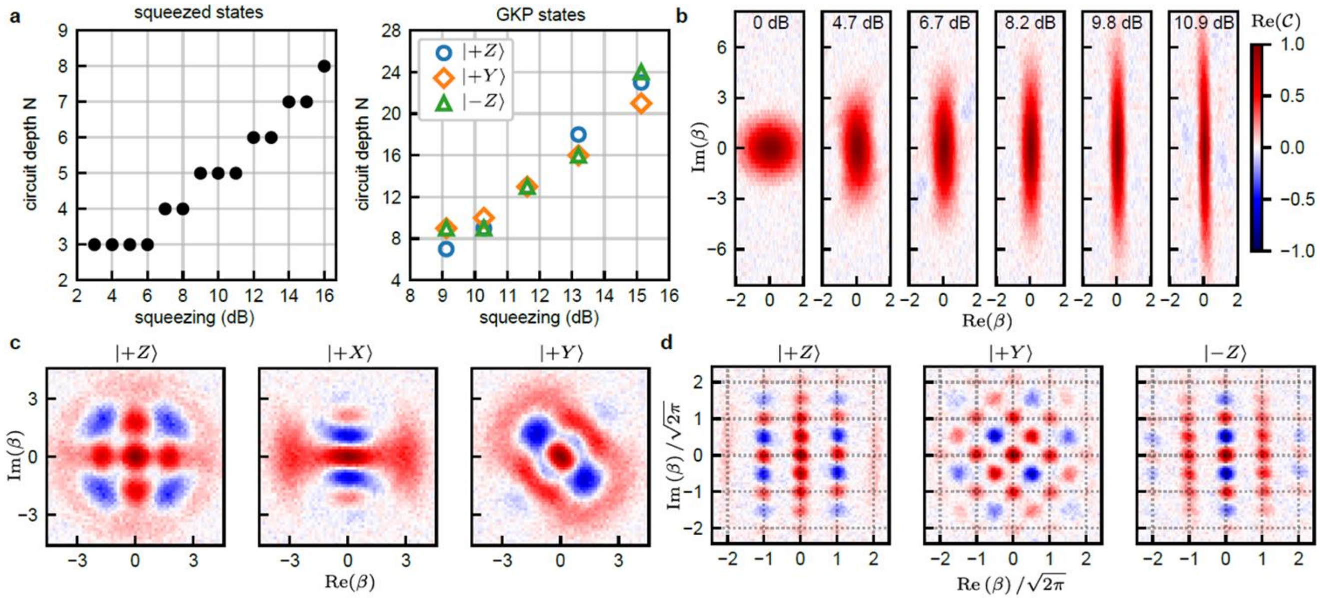
Squeezed states



Binomial QEC code word states

GKP QEC code word states

Numerically optimized gate sequences are highly expressive (efficient) but <u>incomprehensible</u>.



Hybrid CV-DV Architecture

Compiler/Transpiler

Qubit centric | Oscillator centric | Hybrid

Transpiler-visible AMMs

Instruction Set Architecture

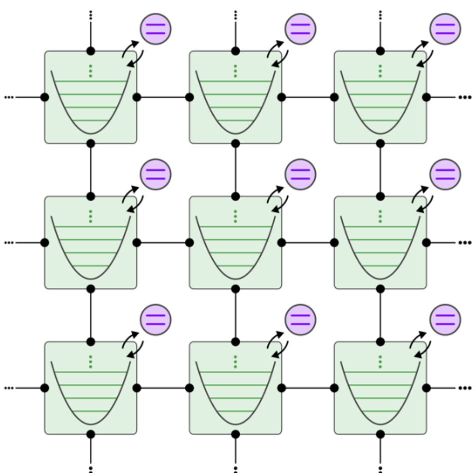
Hybrid Oscillator-Qubit Device

High-level instructions

⇓

Low-level instructions

Connectivity, QEC options, Error model, …



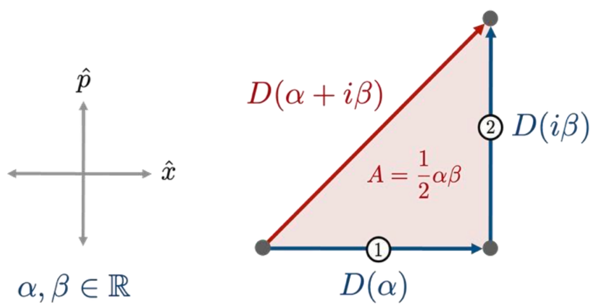
Are there ways to <u>analytically</u> synthesize oscillator-oscillator and multi-qubit gates?

Oscillator-mediated multi-qubit gates (ion traps and circuit QED)

Displacement of oscillator: $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$

Qubit-controlled displacement: $\mathrm{CD}(\alpha) = e^{Z(\alpha a^\dagger - \alpha^* a)}$

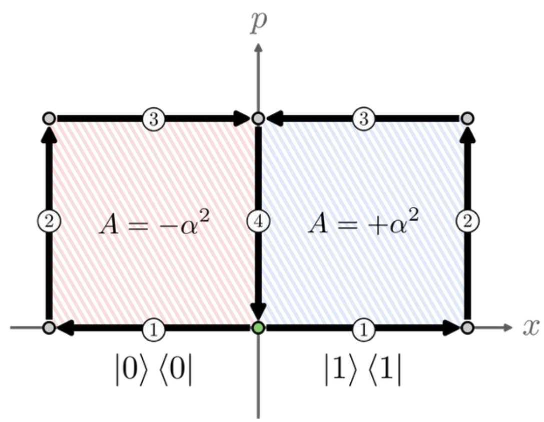
The key ingredient:
Phase space displacements do not commute



$D(i\beta)D(\alpha) = e^{i\alpha\beta}D(\alpha + i\beta)$

A warmup example: $R_Z(\theta) = e^{-i\frac{\theta}{2}Z}$

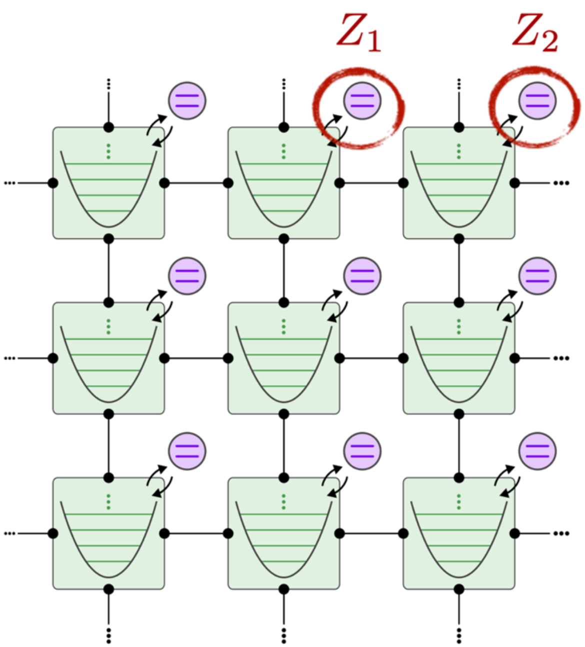
$D(-i\alpha)\mathrm{CD}(\alpha)D(i\alpha)\mathrm{CD}(-\alpha) = e^{i2\alpha^2 Z}$



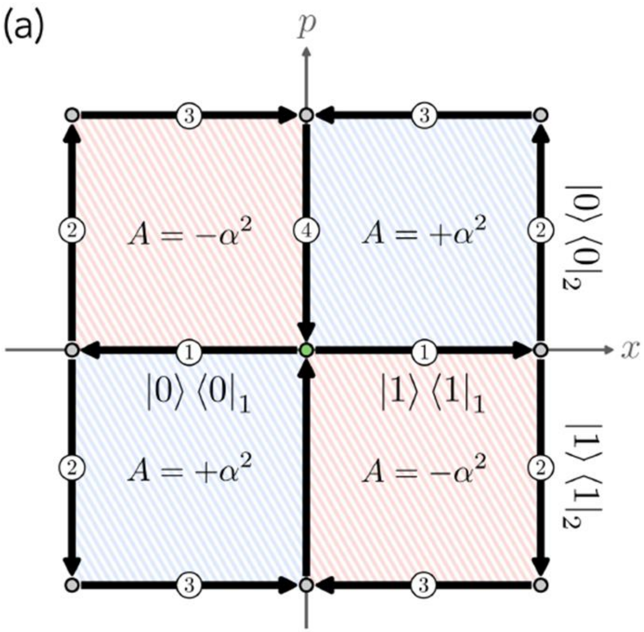
Example Entangling Gate: $ZZ(\theta) = e^{-i\frac{\theta}{2}Z_1 \otimes Z_2}$

$$e^{-2i\alpha^2 Z_1 \otimes Z_2} = \mathrm{CD}_2(-i\alpha)\mathrm{CD}_1(+\alpha)\mathrm{CD}_2(+i\alpha)\mathrm{CD}_1(-\alpha)$$

Note: $\mathrm{CD}_2 = \mathrm{SWAP}_{12}\mathrm{CD}_1\mathrm{SWAP}_{12}$



(a)



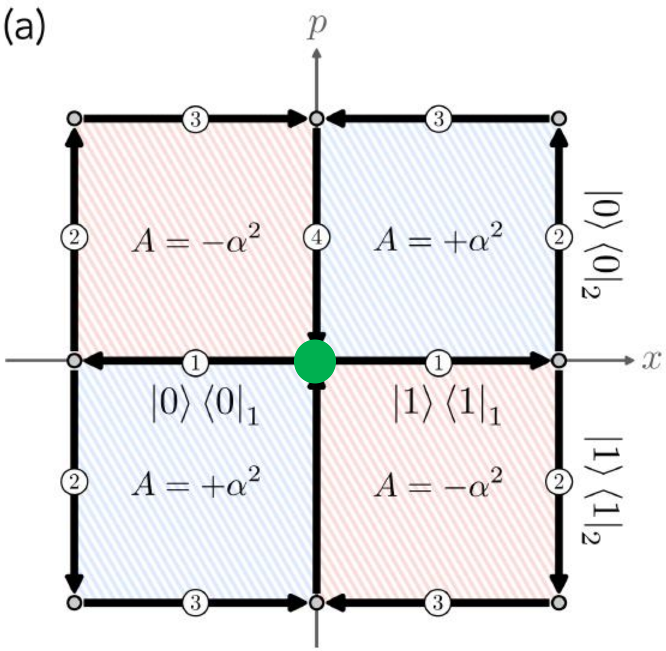
$R_{ZZ}(\theta) = e^{-i\frac{\theta}{2}Z_1 \otimes Z_2}$

4 possible closed loops in phase space:

$Z_1 Z_2 = +1 \Rightarrow$ CW

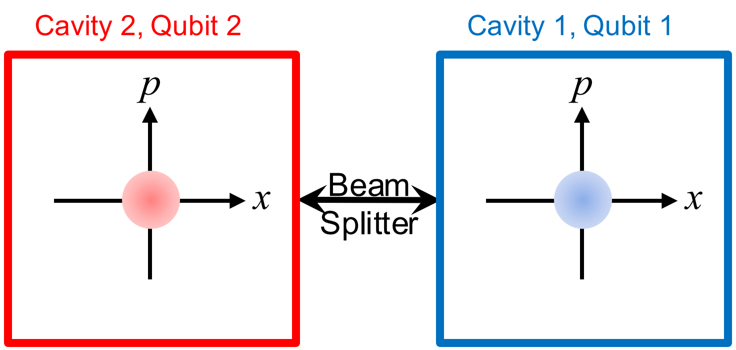
$Z_1 Z_2 = -1 \Rightarrow$ CCW

Example qubit-qubit entangling gate: $ZZ(\theta) = e^{-i\frac{\theta}{2}Z\otimes Z}$



$$R_{ZZ}(\theta) = e^{-i\frac{\theta}{2}Z_1\otimes Z_2}$$

'Movie' for the case $Z_1 = Z_2 = +1$



$$e^{-2i\alpha^2 Z_1\otimes Z_2} = \mathrm{CD}_2(-i\alpha)\mathrm{CD}_1(+\alpha)\mathrm{CD}_2(+i\alpha)\mathrm{CD}_1(-\alpha)$$

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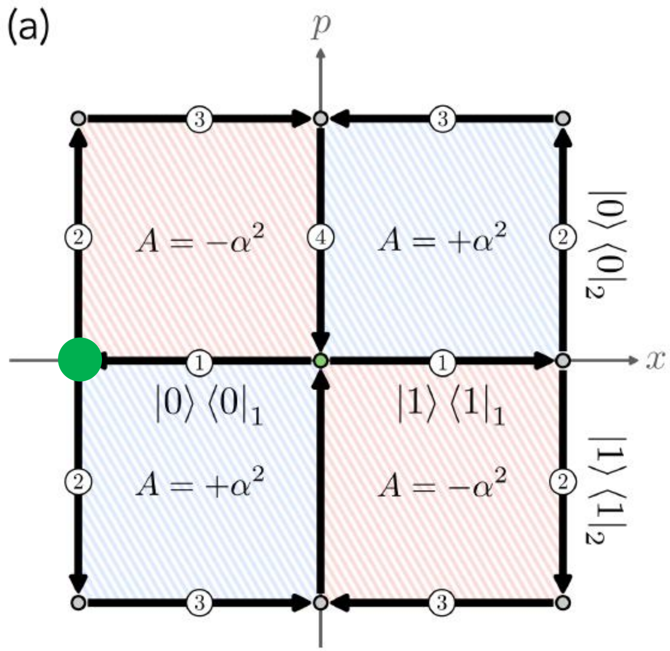

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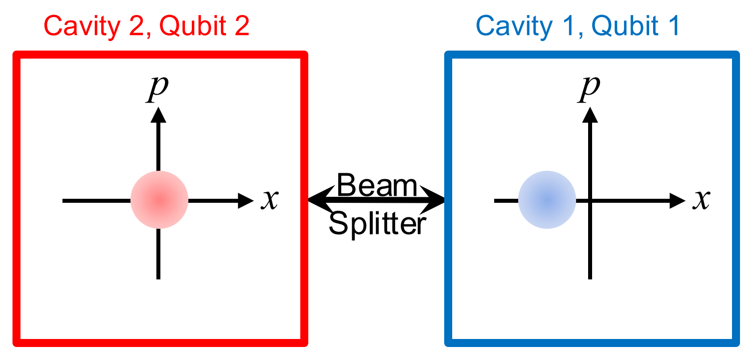

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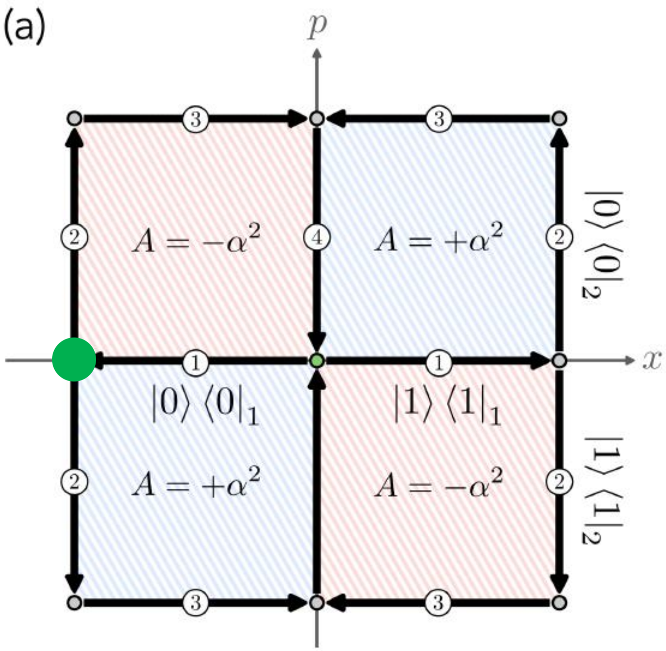

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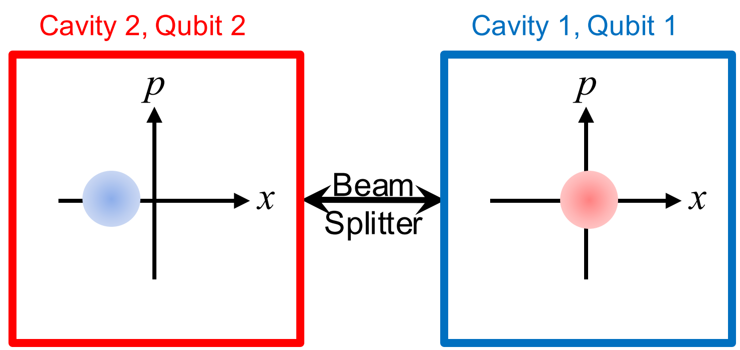

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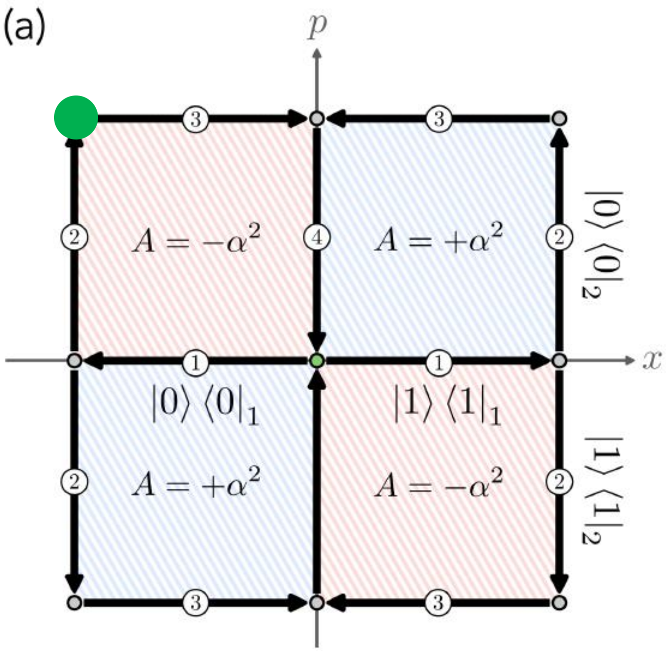

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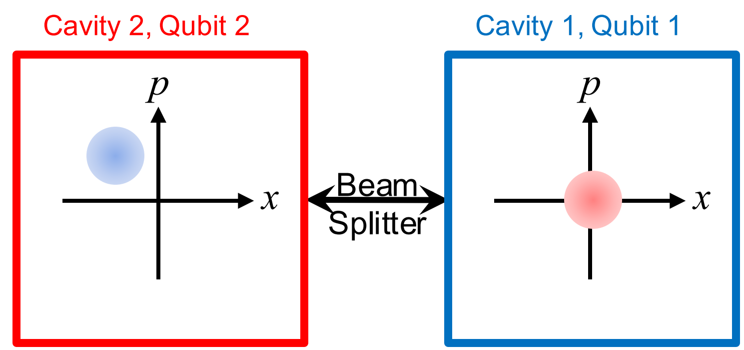

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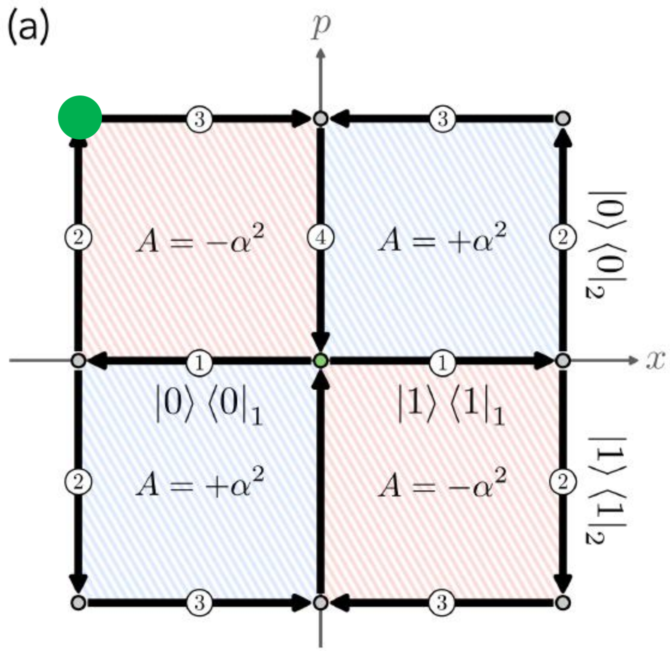

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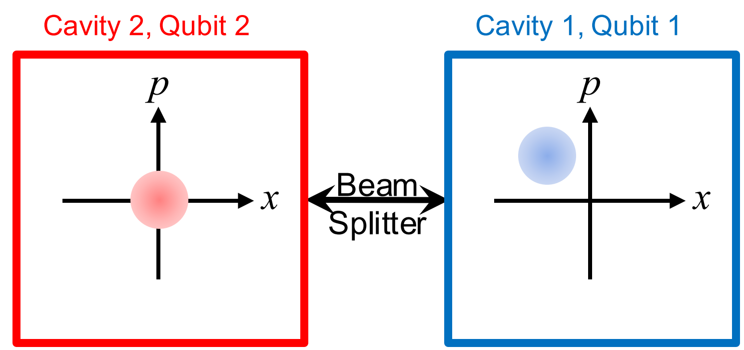

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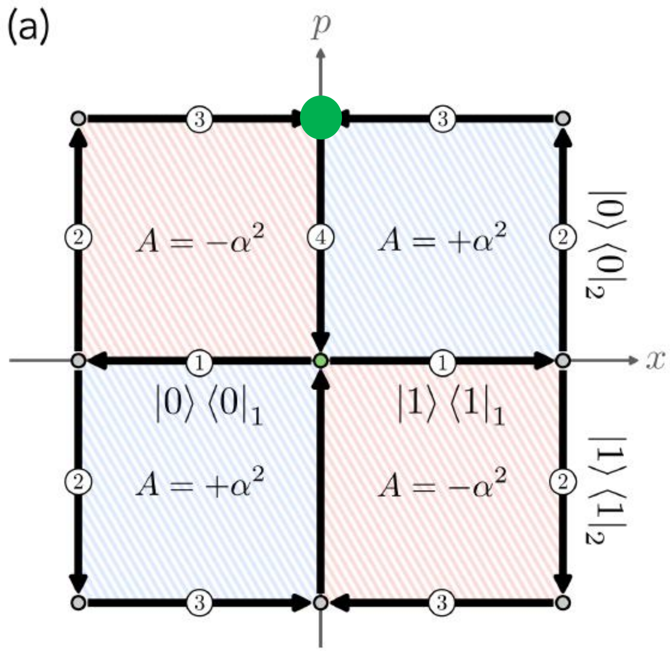

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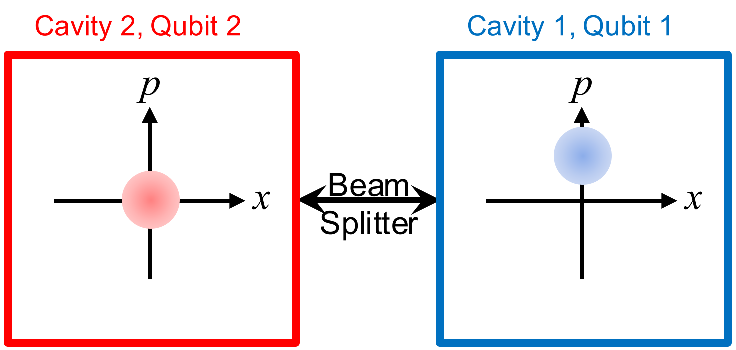

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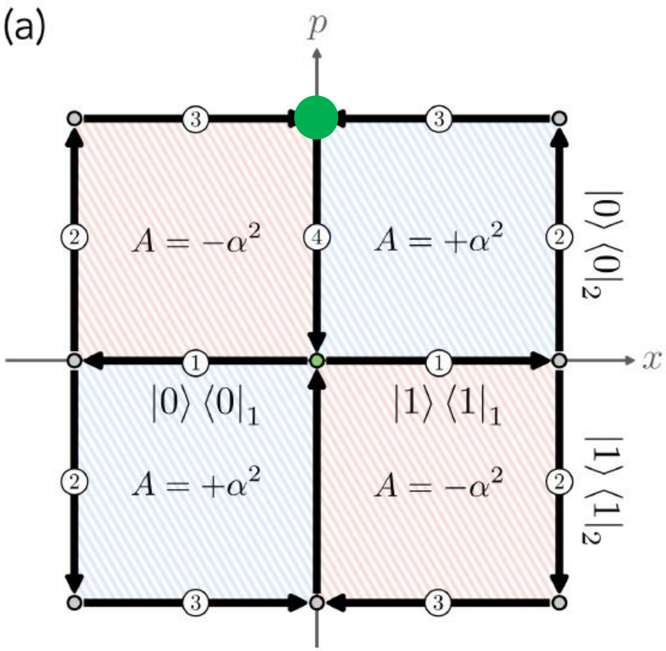

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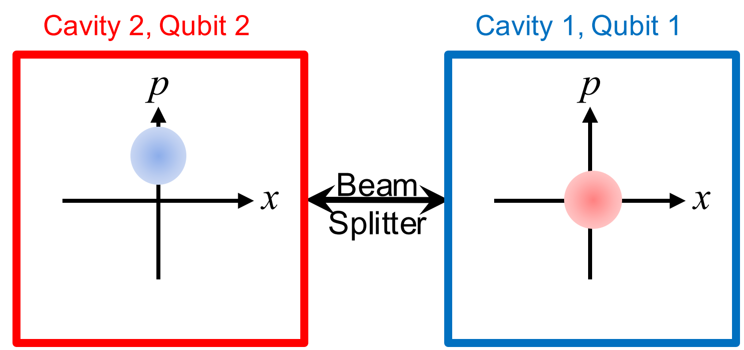

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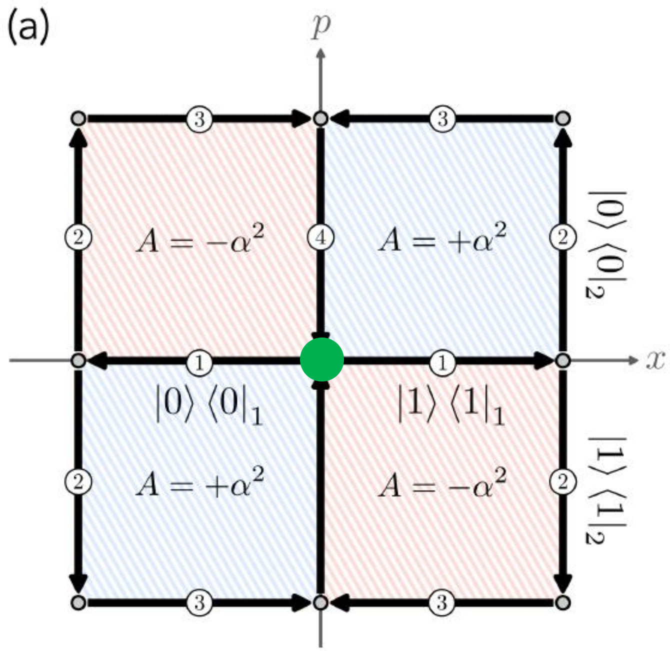

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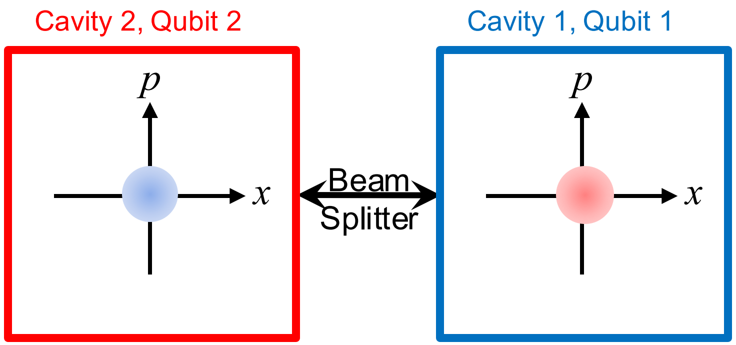

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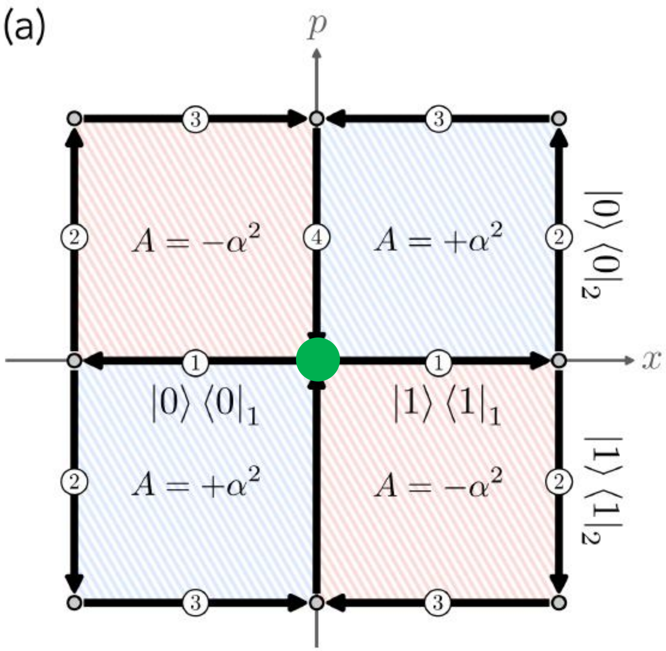

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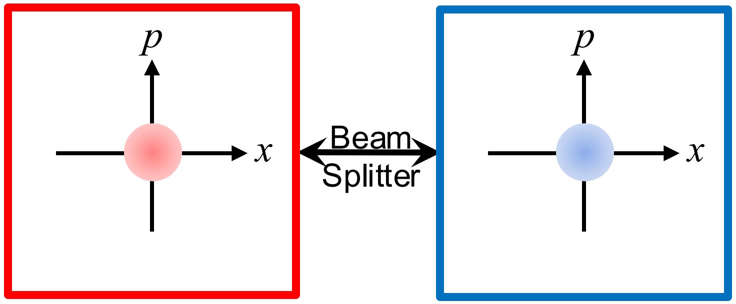

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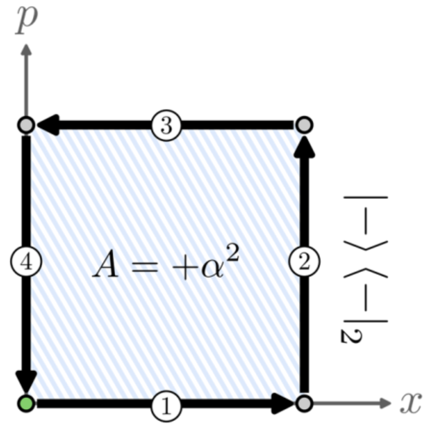
Qubits 1 and 2 can be separated by arbitrary distance
High fidelity SWAPS F=0.999 (0.9998 with post-selection on photon loss)

No dependence on oscillator state!
(Disregarding photon loss, cavity self-Kerr)

More multi-qubit gates

Details in arXiv: 2407.10381

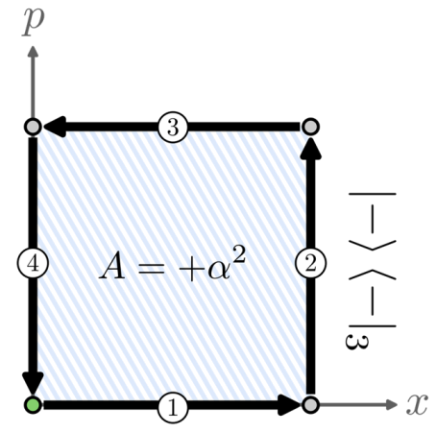
CNOT



$|1\rangle\langle 1|_1$

All other states: $A = 0$

Toffoli



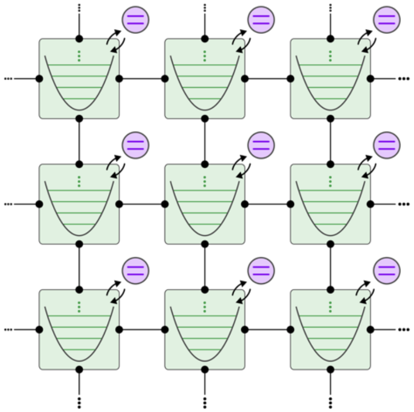
$|1\rangle\langle 1|_1 \otimes |1\rangle\langle 1|_2$

All other states: $A = 0$

Arbitrary Pauli-Weight Multi-Qubit Gates

N-qubit gates: $e^{-i\frac{\theta}{2}P}$ $\quad P = \bigotimes_{i=1}^{N} P_i \quad P_i \in \{I, X, Y, Z\}$

Many interesting open questions



Local Alternating Quantum Classical Computations

LAQCC Computational Model (arXiv:2307.14840)

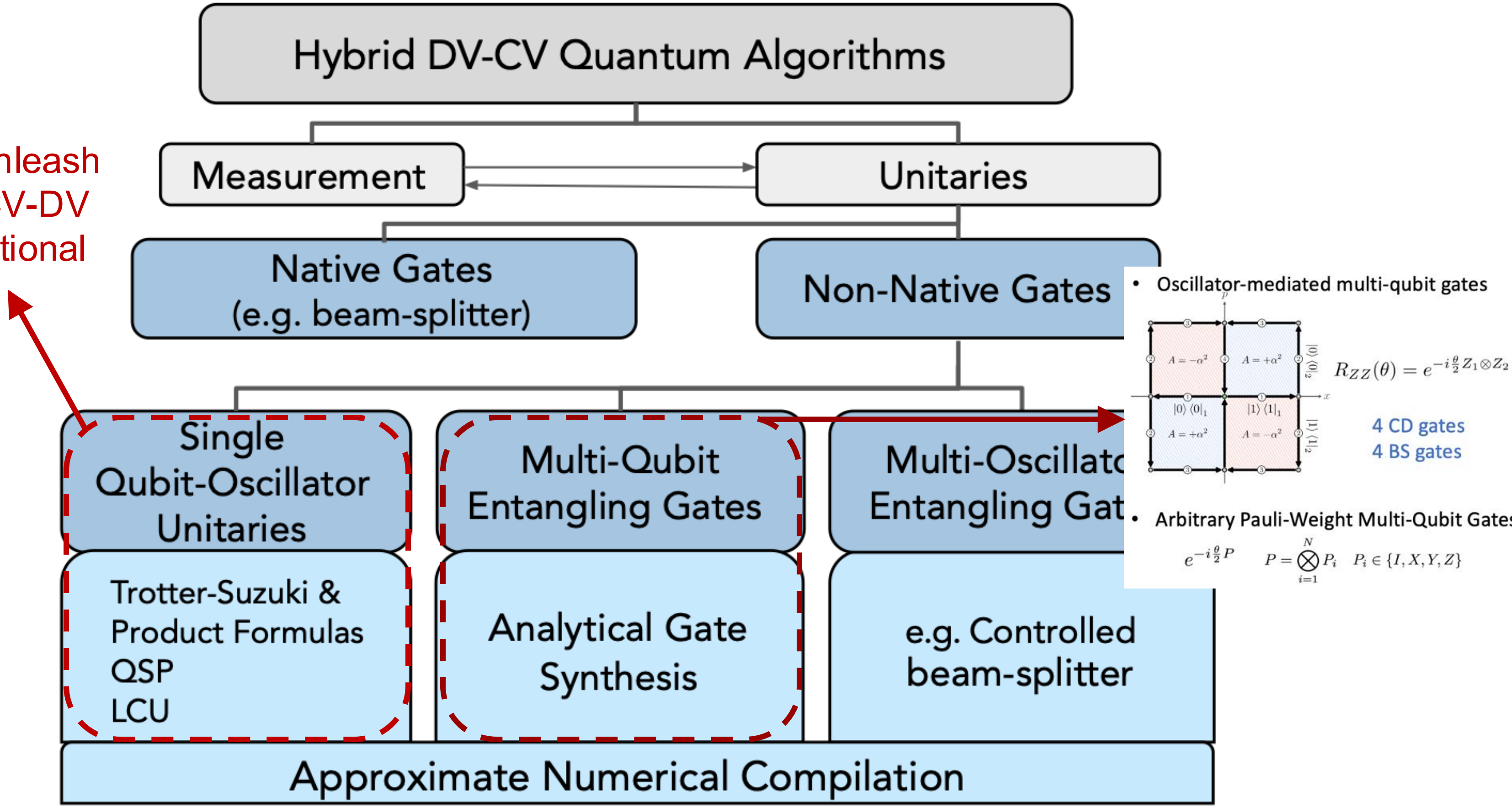
- Mid-circuit measurement and feedforward
  - "Free" qubit ancilla bus
  - GHZ state injection: (constant-time) non-local gates between oscillators

⇨ Quantum simulation with long-range interactions ?

⇨ LDPC error correcting codes with non-local stabilizers ?

⇨ Non-local Multi-mode bosonic codes ?

Key to unleash Hybrid CV-DV computational power!



Two Representations of Hybrid CV-DV Operators

(n qubits, m oscillators)

- **Hermitian Generator Picture**

$$H = \sum_{k=0}^{4^n-1} P_k h_k(\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2, \cdots, \hat{x}_m, \hat{p}_m)$$

P_k : generators for n-qubit system (e.g., Pauli)

e.g., single qubit-oscillator:

$$H = h_0(\hat{x}, \hat{p}) + \vec{h}_1(\hat{x}, \hat{p}) \cdot \vec{\sigma}$$

- **Unitary Picture**

$$U = A_0(\hat{w}_1, \hat{w}_2, \cdots, \hat{w}_m, \hat{v}_1, \hat{v}_2, \cdots, \hat{v}_m) + i \sum_{k=1}^{4^n-1} P_k A_k(\hat{w}_1, \hat{w}_2, \cdots, \hat{w}_m, \hat{v}_1, \hat{v}_2, \cdots, \hat{v}_m)$$

$$w_j = e^{+i2k_j \hat{x}_j}$$

$$\hat{v}_j = e^{-i2\lambda_j \hat{p}_j}$$

Origins of QSP: Robust Spin Rotation Control in NMR

$$R_{\underline{\varphi}}(\underline{\theta}) = e^{-i\frac{\theta}{2}[\cos(\varphi)\sigma_X + \sin(\varphi)\sigma_Y]}$$

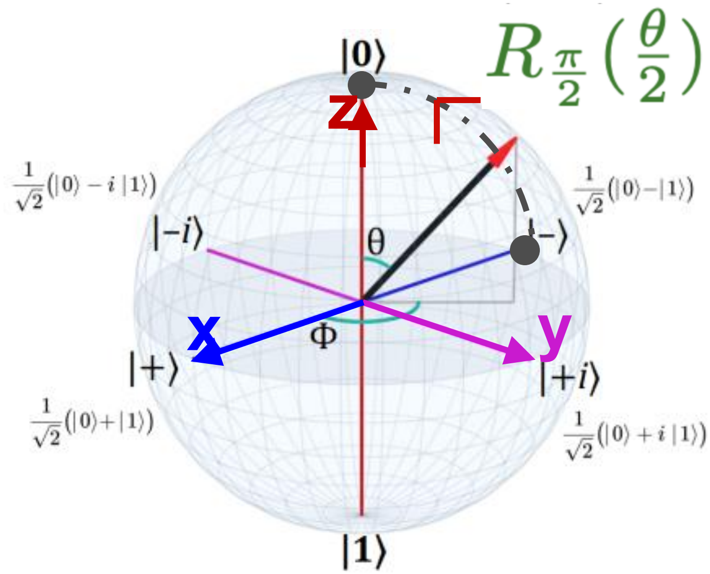
Drive pulse phase　　Drive pulse amplitude

$\theta = \pi/2$ and $\varphi = 0$

Error ⬇ No error on φ

$\theta = (\pi/2)(1 + \underline{\epsilon})$

roughly constant in time



$$\langle \sigma_z \rangle \approx 1 - 1.2\epsilon^2$$

⬇ Error suppression!

$$\langle \sigma_z \rangle \approx 1 - 3.7\epsilon^6$$

Composite pulses:

$$\boxed{\mathrm{BB1}_{90,x} = R_{\varphi_1}(2\theta)R_{\varphi_2}(4\theta)R_{\varphi_1}(2\theta)R_0(\theta)}$$

$$\boxed{\varphi_1 = \cos^{-1}(-1/8),\ \varphi_2 = 3\varphi_1}$$

$$\boxed{\mathrm{BB1}_{90,y} = Z(+\frac{\pi}{2})\mathrm{BB1}_{90,x}Z(-\frac{\pi}{2})}$$

Origins of QSP: Robust Spin Rotation Control in NMR

$$R_{\varphi}(\theta) = e^{-i\frac{\theta}{2}[\cos(\varphi)\sigma_X + \sin(\varphi)\sigma_Y]}$$

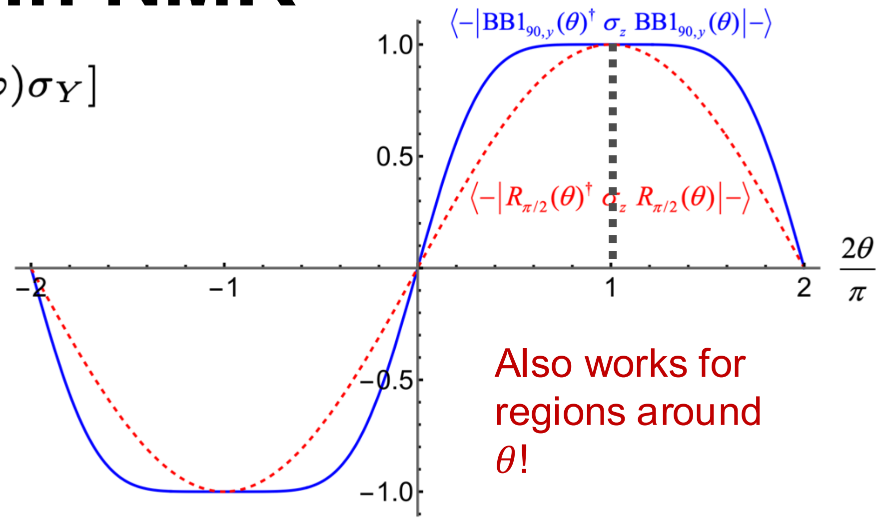
Drive pulse phase (φ) Drive pulse amplitude (θ)

$\theta = \pi/2$ and $\varphi = 0$

Error ⬇ No error on φ

$\theta = (\pi/2)(1+\epsilon)$

roughly constant in time (ϵ)



Also works for regions around θ!

$$\langle \sigma_z \rangle \approx 1 - 1.2\epsilon^2$$

Error suppression!

$$\langle \sigma_z \rangle \approx 1 - 3.7\epsilon^6$$

Composite pulses:

$$\boxed{\mathrm{BB1}_{90,x} = R_{\varphi_1}(2\theta)R_{\varphi_2}(4\theta)R_{\varphi_1}(2\theta)R_0(\theta)}$$

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$$\boxed{\mathrm{BB1}_{90,y} = Z(+\frac{\pi}{2})\mathrm{BB1}_{90,x}Z(-\frac{\pi}{2})}$$

Formal Exhibition of QSP

Composite Pulses

$$\mathrm{BB1}_{90,x} = \boxed{R_{\varphi_1}(2\theta)} R_{\varphi_2}(4\theta) R_{\varphi_1}(2\theta) R_0(\theta)$$

$R_z(-\varphi_1)Rx(2\theta)Rz(\varphi_1)$

$$\mathrm{BB1}_{90,y} = Z(\phi_0) \prod^{d} W_x(\theta) Z(\phi_j)$$

$$\vec{v}_\phi \equiv (\phi_0, \phi_1, \ldots, \phi_d)$$

Degree-9 QSP

$$= \left(\varphi_1 + \frac{\pi}{2}, 0, -\varphi_1 + \varphi_2, 0, 0, 0, -\varphi_2 + \varphi_1, 0, -\varphi_1, -\frac{\pi}{2}\right)$$

$$\mathrm{BB1}_{90,y} = e^{-i\frac{\pi}{2}\vec{h}(\theta)\cdot\vec{\sigma}}$$

$$\vec{h}(\theta) = (h_0(\theta), h_x(\theta), h_y(\theta), h_z(\theta))$$

Quantum Signal Processing (qubit)

Given: Block-encoding of a

$$W(a) = \begin{bmatrix} \boxed{a} & i\sqrt{1-a^2} \\ i\sqrt{1-a^2} & a \end{bmatrix} \qquad S(\phi) = e^{i\phi Z}$$

Signal operator **Signal processing operator**

$\vec{\phi}$ *exists such that*

$$e^{i\phi_0 Z} \prod_{k=1}^{d} W(a) e^{i\phi_k Z}$$

Poly. transformation of a

$$= \begin{bmatrix} \boxed{P(a)} & iQ(a)\sqrt{1-a^2} \\ iQ^*(a)\sqrt{1-a^2} & P^*(a) \end{bmatrix}$$

If and only if:

(i) $deg(P) \leq d$, $deg(Q) \leq d-1$ (expressivity)

(ii) P *has parity* $d \bmod 2$ *and* Q *has parity* $(d-1) \bmod 2$ (parity constraint)

(iii) $|P|^2 + (1-a^2)|Q|^2 = 1$ (unitarity)

More generalizations: QSVT, GQSP, M-QSP, SU(N), SU(1,1), non-linear Fourier transform, …

Low and Chuang. PRL 118, 010501 (2017).

From DV to Hybrid CV-DV QSP

Composite Pulses

$$\mathrm{BB1}_{90,y} = e^{-i\frac{\pi}{2}\vec{h}(\theta)\cdot\vec{\sigma}}$$

$\boldsymbol{\theta \rightarrow (1+\epsilon)\theta}$

Noisy classical variable

Single oscillator-qubit Universal Control

$$H = h_0(\hat{x},\hat{p}) + \vec{h}_1(\hat{x},\hat{p})\cdot\vec{\sigma},$$

Is it possible to promote $\boldsymbol{\theta}$ from a <u>noisy classical variable</u> to a <u>quantum operator $\widehat{\boldsymbol{\theta}}$</u> such as $\hat{x}$ acting on an oscillator? **YES!**

Hybrid CV-DV Quantum Signal Processing

Two views of the oscillator-qubit gate

1) Qubit-dependent momentum boost

$$e^{-i\frac{k}{2}\hat{x}\sigma_z} = \begin{bmatrix} e^{-i\frac{k}{2}\hat{x}} & 0 \\ 0 & e^{i\frac{k}{2}\hat{x}} \end{bmatrix} := W_z$$

Provides a block-encoding of $\hat{x}$ or $\hat{p}$!

2) Oscillator-dependent qubit rotation

$$W_z(\hat{\theta}) = e^{-i\frac{\hat{\theta}}{2}\sigma_z}$$

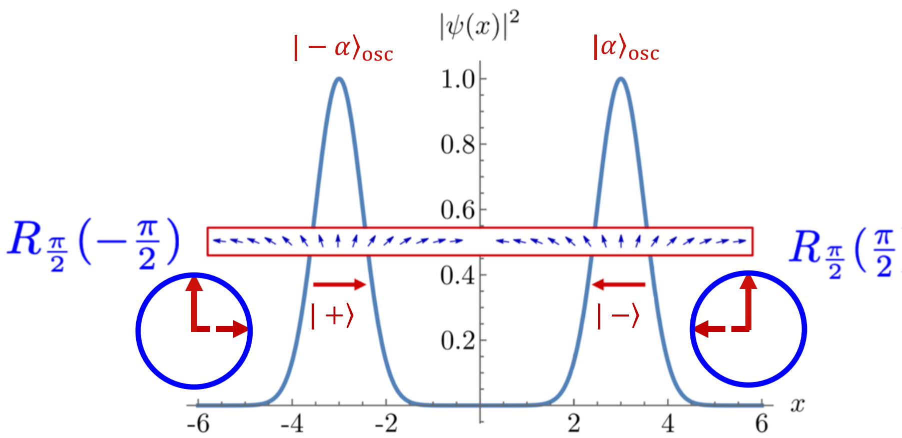
$$\hat{\theta} \equiv k\hat{x}.$$

Quantum fluctuation from the oscillator position plays the role of “noise” on $\widehat{\boldsymbol{\theta}}$!

Example: deterministic preparation of cat state

$$e^{-i2\alpha\hat{p}\otimes\sigma_x}|0\rangle_{\text{vac}} \otimes |0\rangle = \frac{1}{\sqrt{2}}\left[|\alpha\rangle_{\text{osc}}|-\rangle + |-\alpha\rangle_{\text{osc}}|+\rangle\right]$$

Need to disentangle: $\frac{1}{\sqrt{2}}\left[|\alpha\rangle_{\text{osc}} + |-\alpha\rangle_{\text{osc}}\right] \otimes |0\rangle$



Hybrid CV-DV Quantum Signal Processing

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$$e^{-i2\alpha\hat{p}\otimes\sigma_x}\,|0\rangle_{\text{vac}} \otimes |0\rangle = \frac{1}{\sqrt{2}}\left[|\alpha\rangle_{\text{osc}}|-\rangle + |-\alpha\rangle_{\text{osc}}|+\rangle\right]$$

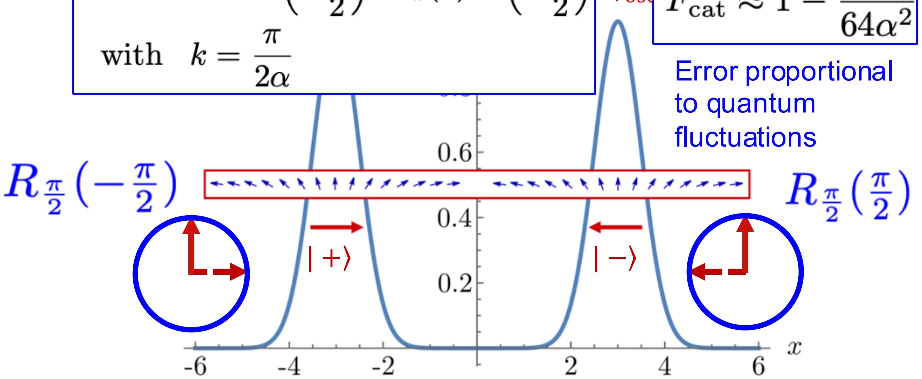
Need to disentangle: $\frac{1}{\sqrt{2}}\left[|\alpha\rangle_{\text{osc}} + |-\alpha\rangle_{\text{osc}}\right] \otimes |0\rangle$

$$e^{-i\frac{k}{2}\hat{x}\otimes\sigma_y} = Z\left(+\frac{\pi}{2}\right) W_x(\hat{\theta})\, Z\left(-\frac{\pi}{2}\right)$$

with $k = \frac{\pi}{2\alpha}$

$$F_{\text{cat}} \approx 1 - \frac{\pi^2}{64\alpha^2}$$

Error proportional to quantum fluctuations



Formal Exhibition of Hybrid CV-DV QSP

- **Qubitization of bosonic mode:**

$$e^{-i\frac{k}{2}\hat{x}\cdot\sigma_z} = \begin{bmatrix} e^{-i\frac{k}{2}\hat{x}} & \\ & e^{i\frac{k}{2}\hat{x}} \end{bmatrix} = \begin{bmatrix} w(\hat{x}) & \\ & w^{-1}(\hat{x}) \end{bmatrix} := W_z^{(k)}(w)$$

Block-encoding of x-quadrature

$$e^{-i\frac{\lambda}{2}\hat{p}\cdot\sigma_z} = \begin{bmatrix} e^{-i\frac{\lambda}{2}\hat{p}} & \\ & e^{i\frac{\lambda}{2}\hat{p}} \end{bmatrix} = \begin{bmatrix} v(\hat{p}) & \\ & v^{-1}(\hat{p}) \end{bmatrix} := W_z^{(\lambda)}(v),$$

Block-encoding of p-quadrature

- **Single-variate (1D filter):**

$$e^{i\phi_0\sigma_x}\prod_{j=1}^{d} W_z e^{i\phi_j\sigma_x} = \begin{bmatrix} F(w) & iG(w) \\ iG(w^{-1}) & F(w^{-1}) \end{bmatrix}$$

$F(w)$ and $G(w)$ satisfy similar constraints as qubit QSP.

$$F(w) = \sum_{n=-d}^{d} f_n w^n = \sum_{n=-d}^{d} f_n e^{-i\frac{nk}{2}\hat{x}} := f(\hat{x}),$$

$$G(w) = \sum_{n=-d}^{d} g_n w^n = \sum_{n=-d}^{d} g_n e^{-i\frac{nk}{2}\hat{x}} := g(\hat{x}).$$

- Phase space filters!
- Many generalizations apply: GQSP, QSVT…

- **Non-abelian bivariate (2D filter):**

$$e^{i\phi_0\sigma_x}\prod_{j=1}^{d} W_z^{(k)} e^{i\phi_j^{(k)}\sigma_x} W_z^{(\lambda)} e^{i\phi_j^{(\lambda)}\sigma_x}$$

$$= \begin{bmatrix} F_d(w,v) & iG_d(w,v) \\ iG_d(v^{-1},w^{-1}) & F_d(v^{-1},w^{-1}) \end{bmatrix}$$

$$F_d(w,v) := \sum_{r,s=-d}^{d} f_{rs} w^r v^s,$$

$$G_d(w,v) := \sum_{r,s=-d}^{d} g_{rs} w^r v^s,$$

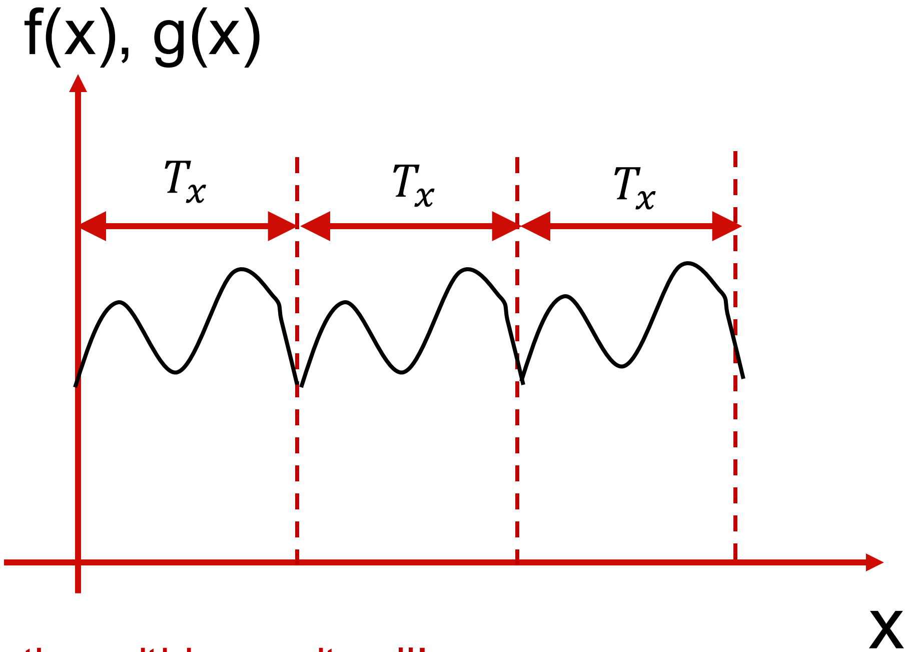
Periodicity of Hybrid CV-DV QSP Transform

$$F(\omega) = \sum_{n=-d}^{d} f_n e^{i\kappa\hat{x}n} := f(\hat{x}),$$

$$G(1/\omega) = \sum_{n=-d}^{d} g_n e^{i\kappa\hat{x}n} := g(\hat{x}),$$

$$f(\hat{x} + mT_x) = f(\hat{x}),$$
$$g(\hat{x} + mT_x) = g(\hat{x}).$$

$$T_x = \frac{2\pi}{\kappa}$$



Approximate a function within a unit cell!

Arbitrarily Accurate Cat State Preparation

Non-abelian QSP: $$F_{\rm cat} \approx 1 - \frac{\pi^2}{64\alpha^2}$$

- Can this be systematically improved? Yes!

$$\mathrm{BB1}_{90,y} = Z(\phi_0)\prod_{j=1}^{d} W_x(\theta) Z(\phi_j)$$

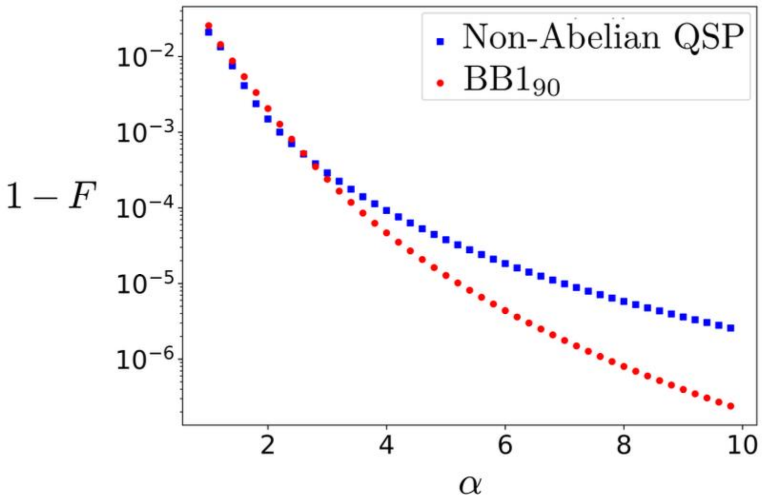
$$e^{-i\frac{k}{2}\hat{x}\cdot\sigma_z} = \begin{bmatrix} e^{-i\frac{k}{2}\hat{x}} & \\ & e^{i\frac{k}{2}\hat{x}} \end{bmatrix} := W_z$$

$$W_z(\hat{\theta}) = e^{-i\frac{\hat{\theta}}{2}\sigma_z}$$
$$\hat{\theta} \equiv k\hat{x}.$$

$$\vec{v}_\phi \equiv (\phi_0, \phi_1, \ldots, \phi_d)$$
$$= \left(\varphi_1 + \frac{\pi}{2}, 0, -\varphi_1 + \varphi_2, 0, 0, 0, -\varphi_2 + \varphi_1, 0, -\varphi_1, -\frac{\pi}{2}\right)$$

$$\varphi_1 = \cos^{-1}(-1/8),\ \varphi_2 = 3\varphi_1$$

Fidelity comparison between $\mathrm{BB1}_{90}$ and non-abelian QSP



Compilation Schemes for Bosonic QEC

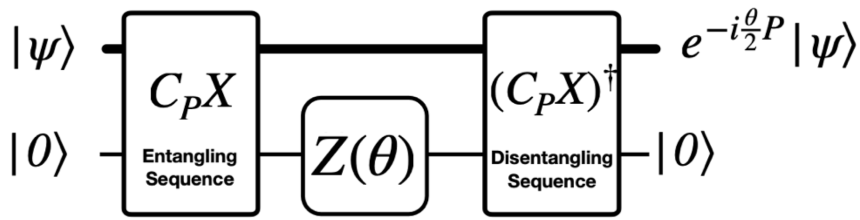
- **Logical readout:**

Entangling the oscillator logical state with ancilla qubit + measure qubit. e.g.,

$$|0\rangle_L |0\rangle + |1\rangle_L |1\rangle$$

How to get entangling operation for the cat code?
- Simply reverse the disentangling process.

- **Logical single-qubit operation:**



- **Codeword stabilization:**

$$|C_{0,\alpha,\pm}\rangle = |\alpha\rangle_{\text{osc}} \pm |-\alpha\rangle_{\text{osc}}$$

Photon loss will shrink the cat size, need to repump energy.

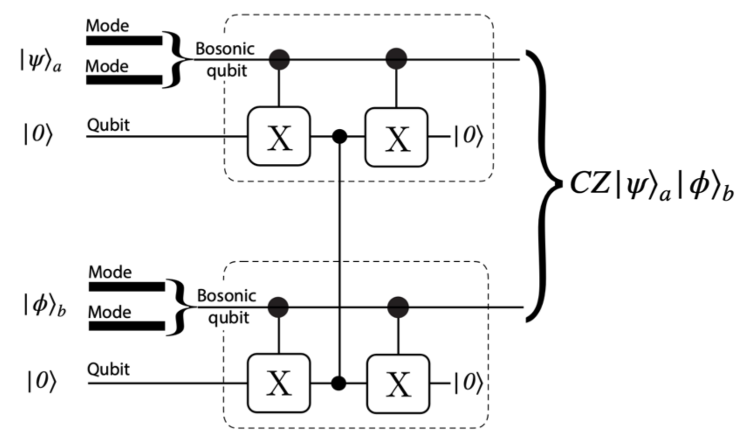
Re-entangle: $\mathcal{E}_x\left[-\frac{\pi}{4\alpha}, 1\right] |C_{0,\alpha,\pm}\rangle |+\rangle = |\alpha\rangle_{\text{osc}} |0\rangle \pm |-\alpha\rangle_{\text{osc}} |1\rangle$

Displace to repump energy:

$$e^{-i\delta\sqrt{2}\hat{p}\sigma_z} |\alpha\rangle_{\text{osc}} |0\rangle \pm |-\alpha\rangle_{\text{osc}} |1\rangle = |\alpha+\delta\rangle_{\text{osc}} |0\rangle \pm |-\alpha-\delta\rangle_{\text{osc}} |1\rangle$$

Unentangle: $\mathcal{U}_x\left[\frac{\pi}{4\alpha'}, 1\right] |\alpha'\rangle_{\text{osc}} |0\rangle \pm |-\alpha'\rangle_{\text{osc}} |1\rangle = |C_{0,\alpha',\pm}\rangle |+\rangle$

- **Entangling two logical bosonic qubits:**

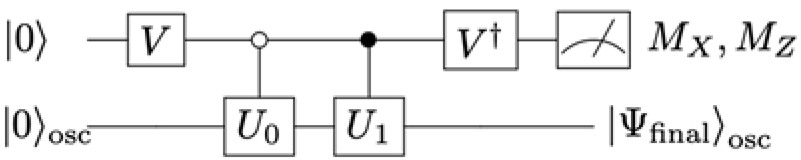


More applications to bosonic QEC: arXiv:2504.19992.

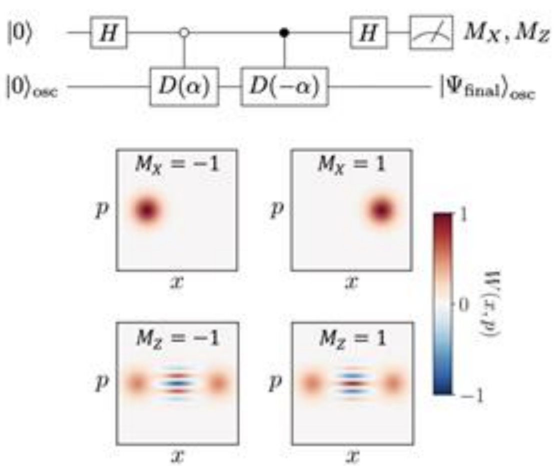
Linear Combination of Unitaries

Linear Combination of CV Unitaries

(**DV** as ancilla)

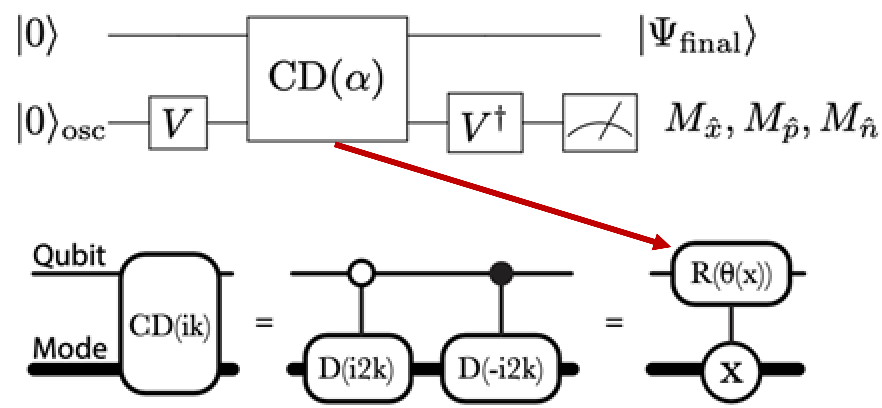


- Example of LCU to Prepare Cat State



Linear Combination of DV Unitaries

(**CV** as ancilla)



$$\boxed{|\psi_{\text{final}}\rangle = \int_{-\infty}^{\infty} dx \; c_{x''}(x) \; \hat{R}(x) \, |0\rangle}$$

Where $c_{x''}(x) = \Psi(x) \langle x''| V^\dagger |x\rangle$ for $\Psi(x) = \langle x| V |0\rangle_{osc}$

Also see Quantum 8, 1496 (2024).

Trotter and Product Formulas

- BCH: $e^{itA}e^{itB}e^{-itA}e^{-itB} = e^{-t^2[A,B]} + O(t^3).$

- Commutation between Pauli gives product of oscillator Hamiltonians.
- Operator norm being unbounded, but **norm with respect to finite-energy states** is still bounded.

- Compilation of Polynomial Product:

$$e^{-i[h_1(\hat{x},\hat{p})h_2(\hat{x},\hat{p})+h_2(\hat{x},\hat{p})h_1(\hat{x},\hat{p})]\otimes\sigma_z}$$

$$= \left(e^{-ih_1\otimes\sigma_x/\sqrt{r}}e^{-ih_2\otimes\sigma_y/\sqrt{r}}e^{ih_1\otimes\sigma_x/\sqrt{r}}e^{ih_2\otimes\sigma_y/\sqrt{r}}\right)^r$$

$$+ O(\max(\|h_1\|,\|h_2\|)^2\min(\|h_1\|,\|h_2\|)/\sqrt{r}). \quad (322)$$

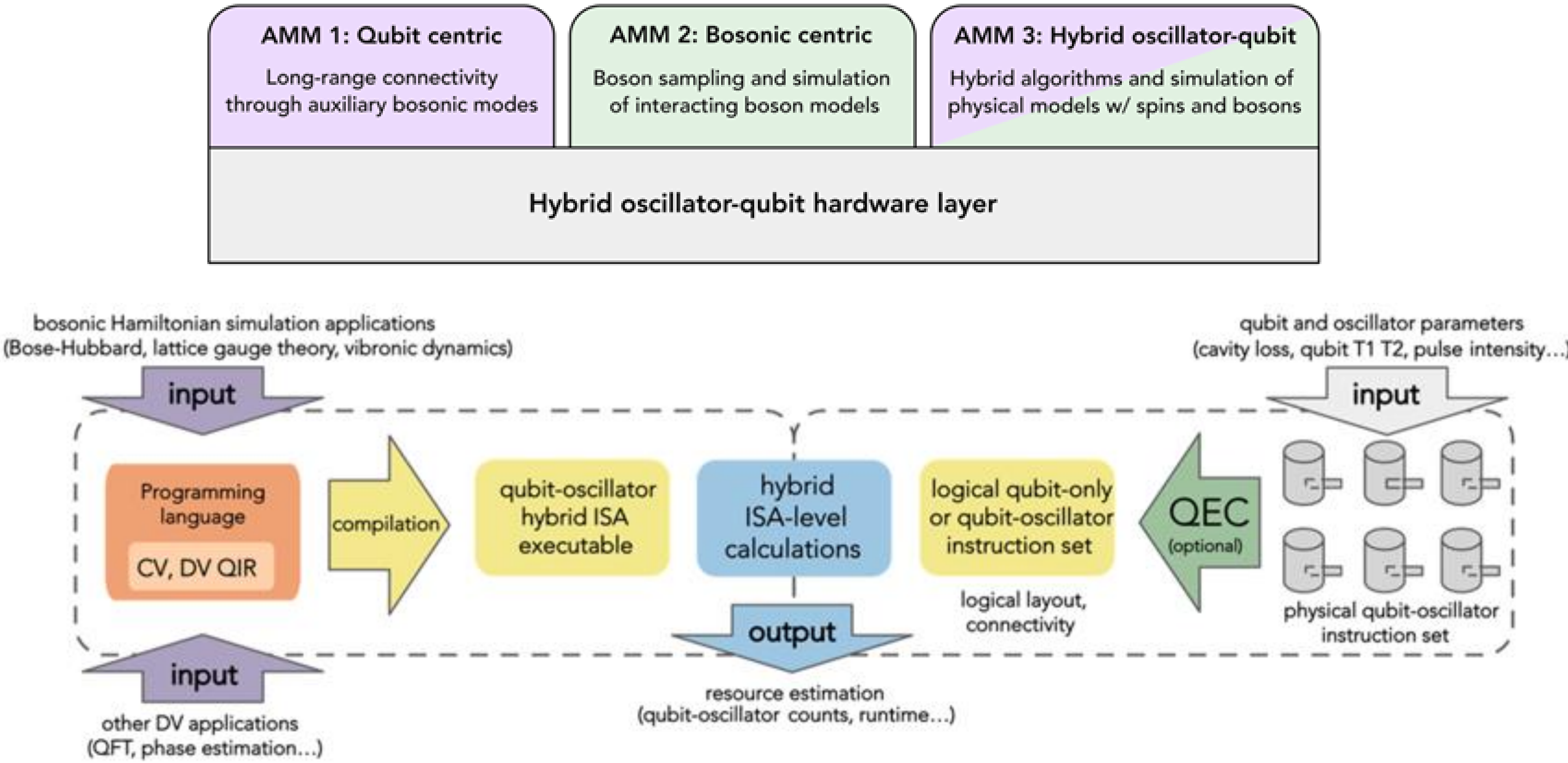
- Example $A = \hat{p}\sigma_y$, $B = \hat{x}\sigma_x$

$$e^{i\sqrt{\zeta}A}e^{i\sqrt{\zeta}B}e^{-i\sqrt{\zeta}A}e^{-i\sqrt{\zeta}B} \approx e^{-\zeta[A,B]}$$

$$\approx e^{i\zeta Z(\hat{x}\hat{p}+\hat{p}\hat{x})}$$

Also see C. Kang, M.B. Soley et al., Kang, Christopher, et al. *Journal of Physics A: Mathematical and Theoretical* (2025).

Resource Estimation on Hybrid Processors

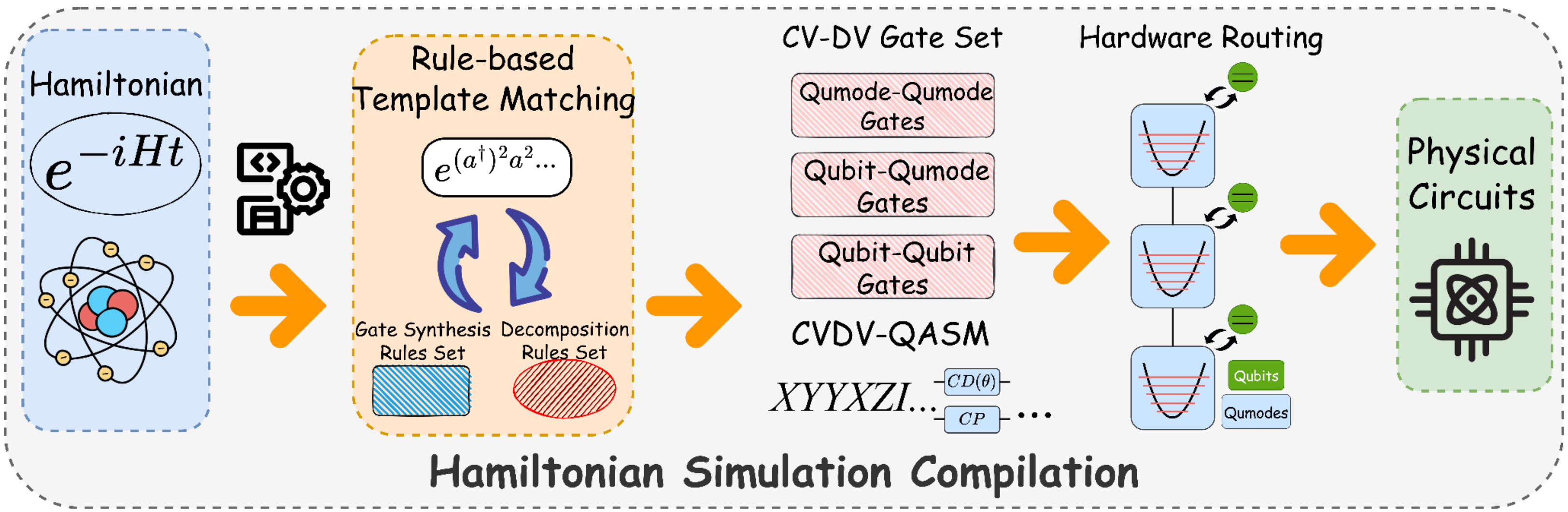


Wednesday, June 25, Session 8B: Quantum III

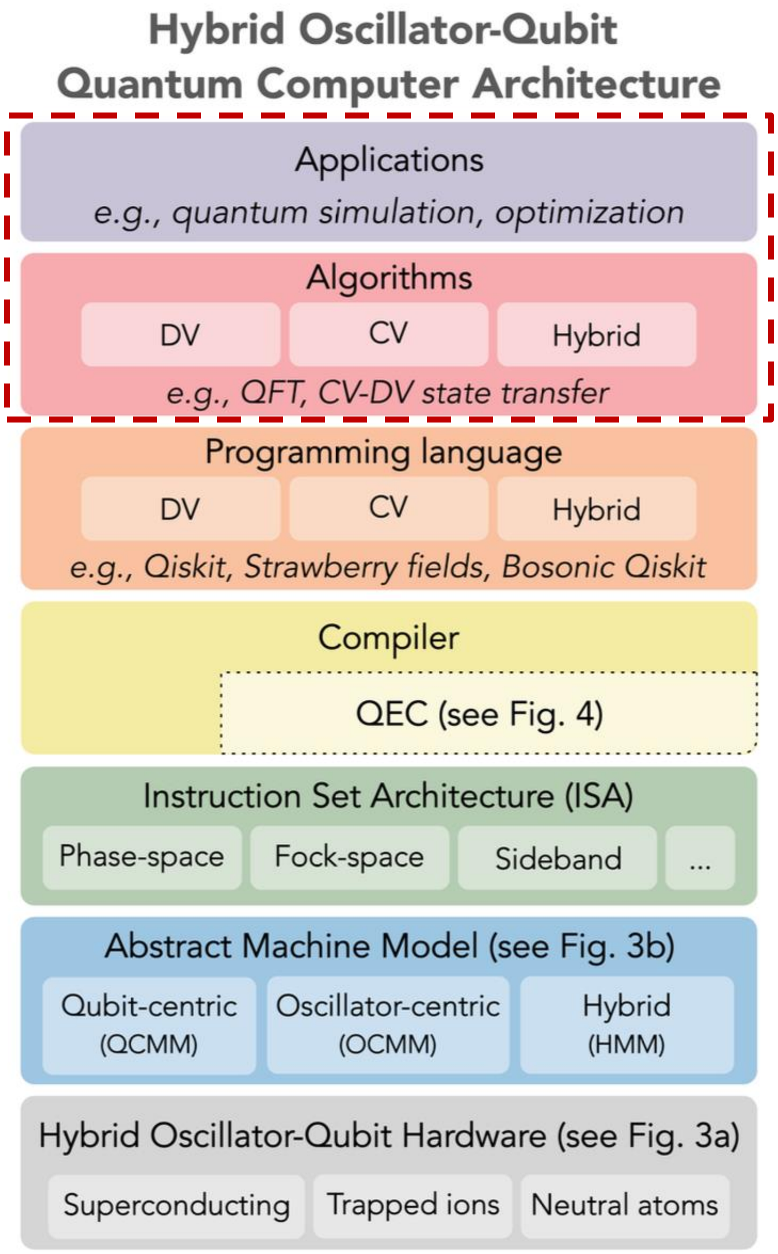
11:00 AM – 11:20 AM

Genesis: A Compiler for Hamiltonian Simulation on Hybrid CV-DV Quantum Computers

Zihan Chen, Jiakang Li, Minghao Guo, Henry Chen, Zirui Li, Joel Bierman, Yipeng Huang, Huiyang Zhou, Yuan Liu, Eddy Z. Zhang



https://github.com/ruadapt/Genesis-CVDV-Compiler



PART III - Algorithms and Applications

- What useful things can we do? Are there advantages?
  - Application I: CV-DV State Transfer
  - Application II: Quantum Fourier Transform
  - Other Applications

Application I: DV-CV State Transfer

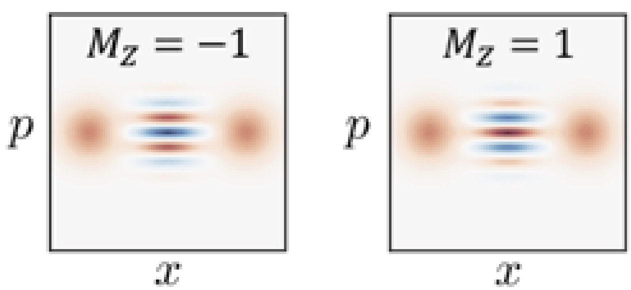
Problem Statement of DV-CV State Transfer

- Given *n*-qubit state $|\psi\rangle_Q = \sum_{\mathbf{x}} c_{\mathbf{x}} |\mathbf{x}\rangle_Q$
- Find unitary operation $U_{\text{st}}^{(n)}(\Delta)$, such that:

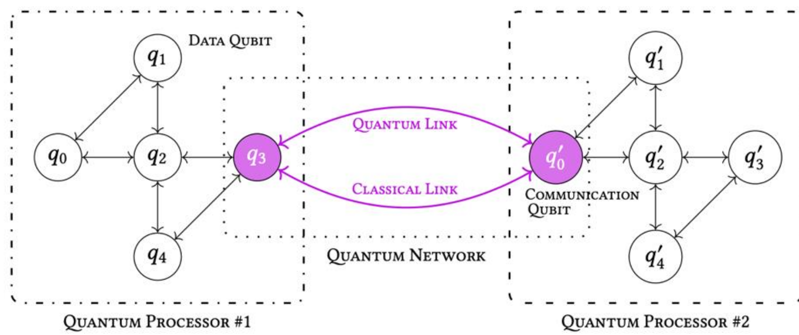
$$U_{\text{st}}^{(n)}(\Delta)|\psi\rangle_Q|0,\Delta\rangle_B = |\mathbf{0}\rangle_Q|\psi\rangle_B$$

- CV-DV transfer follows by simple inverse.

Logical state preparation

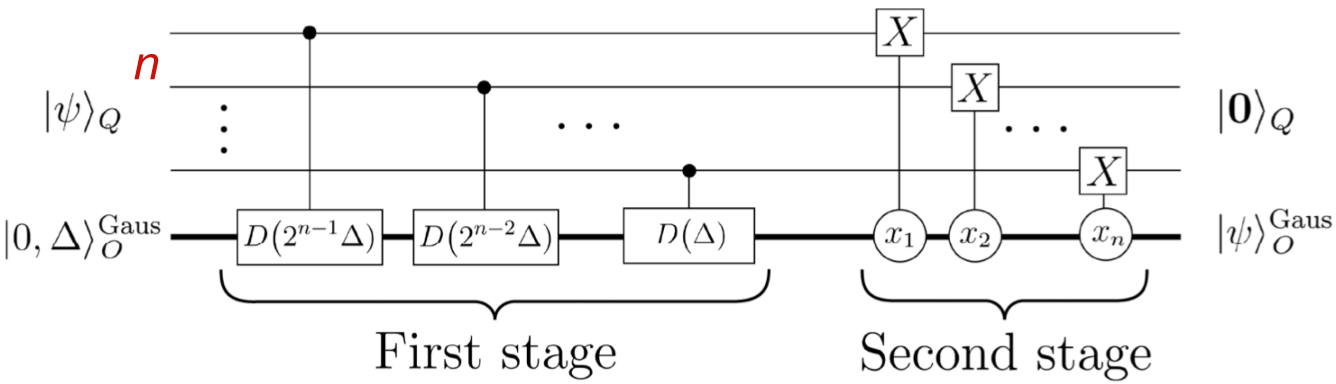


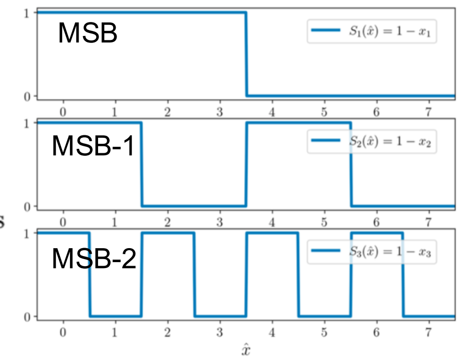
Local quantum computation and quantum networks

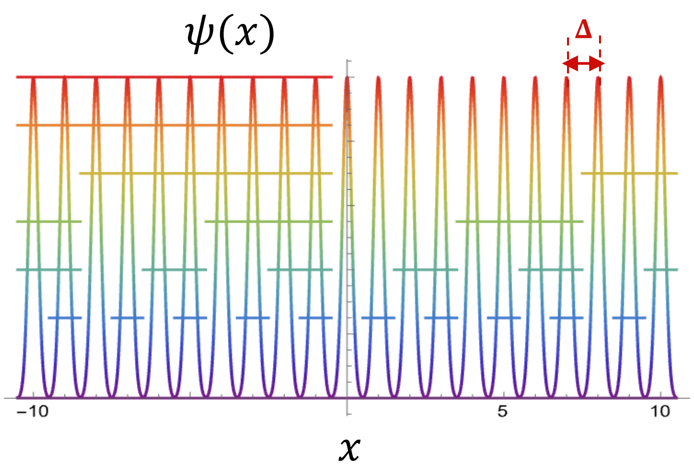


- Depends on how DV computational basis is mapped to CV
- Better to be deterministic
- Need to work for arbitrary states

State Transfer from Single-variable QSP







Fidelity: $1 - \mathcal{O}(n\epsilon) - \mathcal{O}(e^{-\mathcal{O}(\Delta^2/\sigma^2)})$

Gate complexity: $\mathcal{O}(2^n \log(1/\epsilon))$

Time complexity: $\mathcal{O}(2^n(\Delta + \log(1/\epsilon)))$

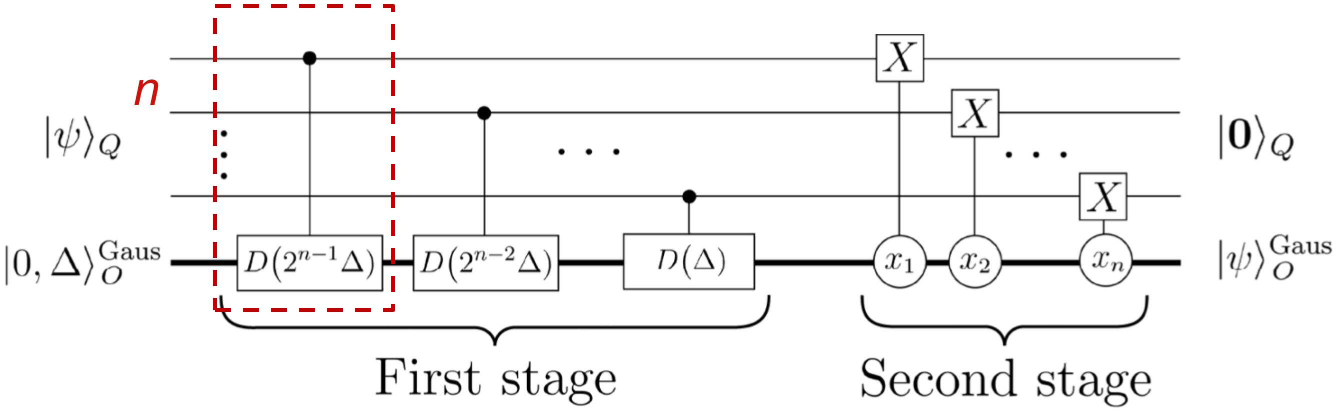
$\boldsymbol{n}$: number of qubits

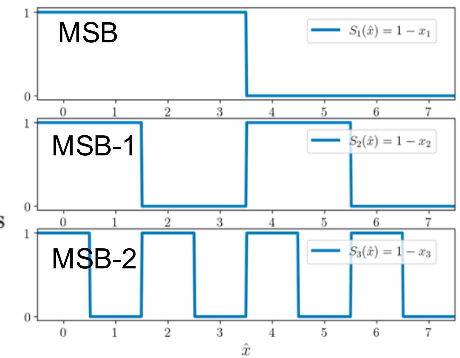
$\boldsymbol{\epsilon}$: error for QSP polynomials

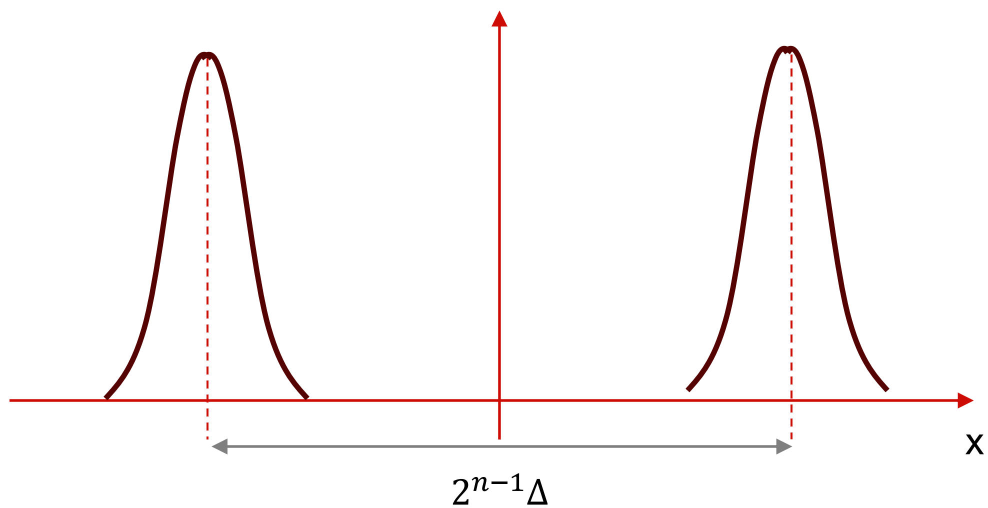
$\boldsymbol{\sigma}$: width of initial Gaussian

Δ: spacing between Gaussians

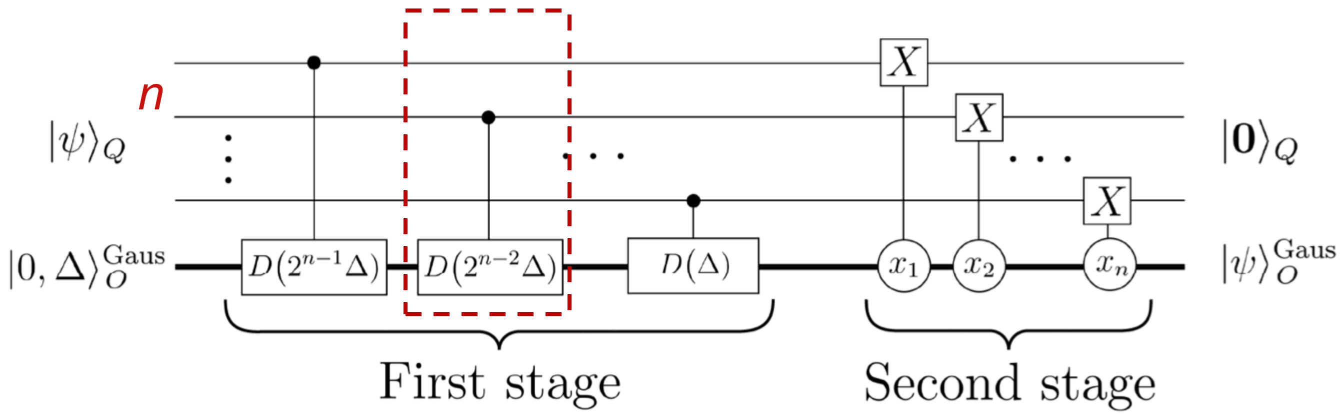
State Transfer from Single-variable QSP

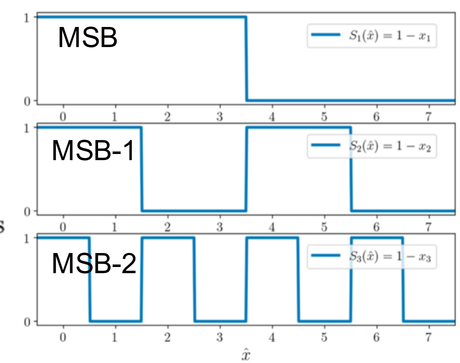


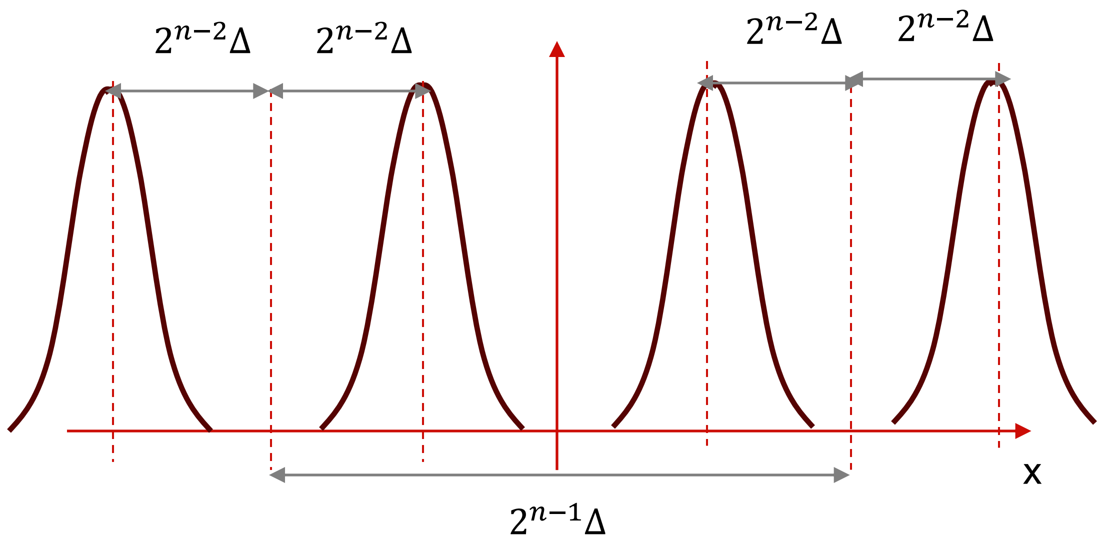




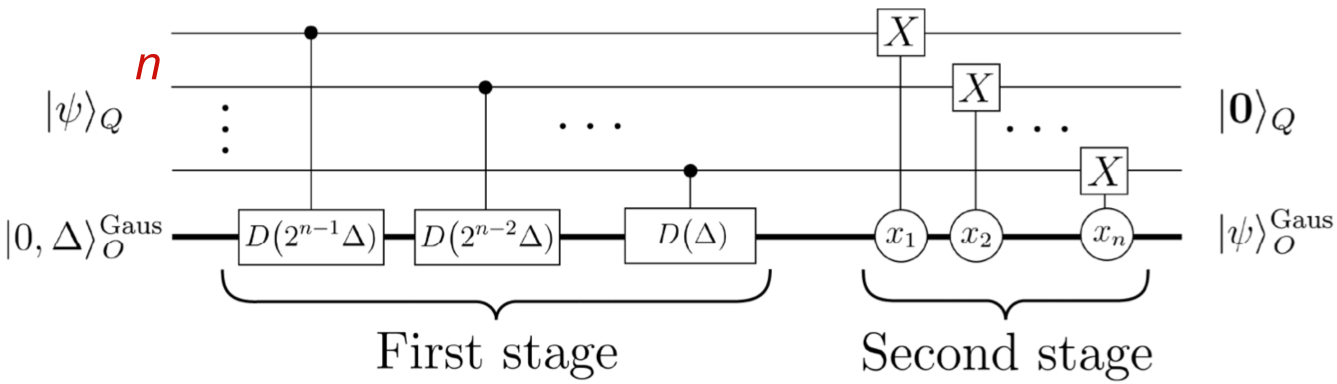
State Transfer from Single-variable QSP

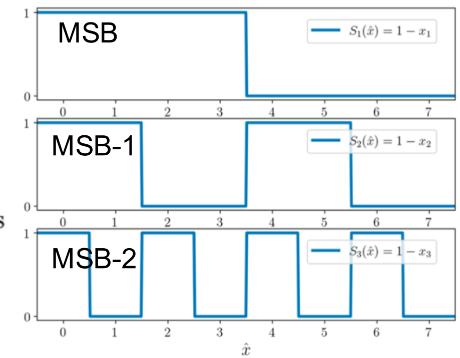


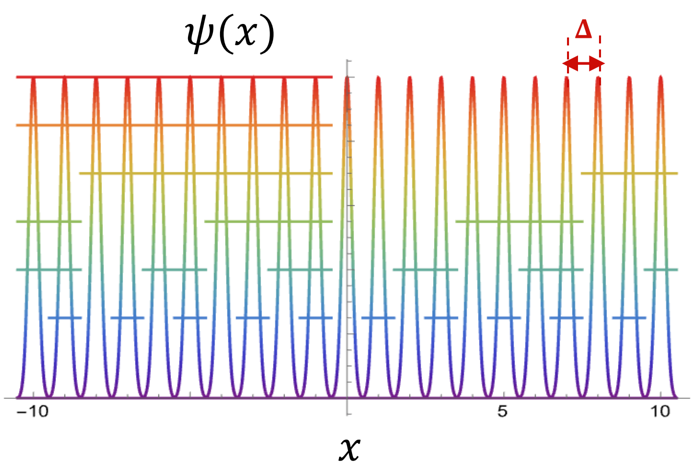




State Transfer from Single-variable QSP







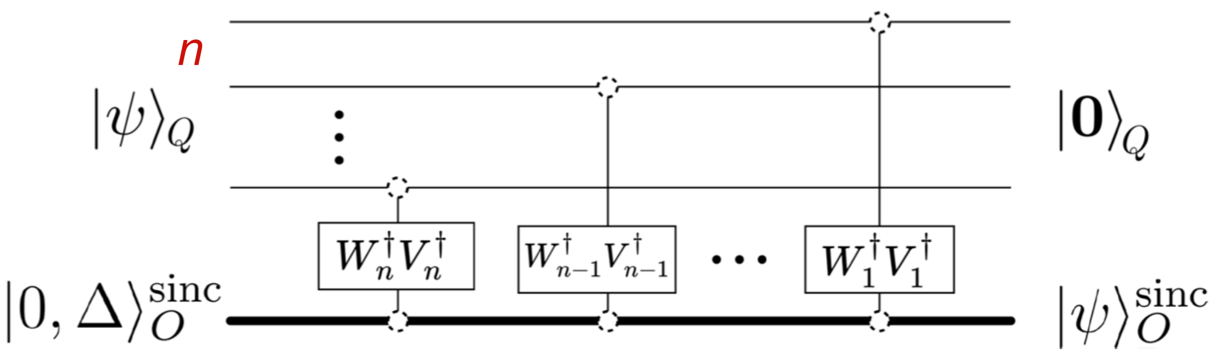
Fidelity: $1 - \mathcal{O}(n\epsilon) - \mathcal{O}(e^{-\mathcal{O}(\Delta^2/\sigma^2)})$

Gate complexity: $\mathcal{O}(2^n \log(1/\epsilon))$

Time complexity: $\mathcal{O}(2^n(\Delta + \log(1/\epsilon)))$

$\boldsymbol{n}$: number of qubits $\boldsymbol{\epsilon}$: error for QSP polynomials
$\boldsymbol{\sigma}$: width of initial Gaussian Δ: spacing between Gaussians

State Transfer from Non-abelian QSP



$$V_j = e^{i\frac{\pi}{2^{j+1}\lambda}\hat{x}\hat{\sigma}_y^{(j)}}$$

$$W_j = \begin{cases} e^{i\lambda 2^{j-1}\hat{p}\hat{\sigma}_x^{(j)}} & j < n \\ e^{-i\lambda 2^{j-1}\hat{p}\hat{\sigma}_x^{(j)}} & j = n \end{cases}$$

Fidelity: $1 - \mathcal{O}\Big(\int_{-\infty}^{-\lambda(2^n-1)} dq|\psi(q)|^2 + \int_{\lambda(2^n-1)}^{\infty} dq|\psi(q)|^2\Big)$

Gate complexity: $\mathcal{O}(n)$

Time complexity: $\boxed{\mathcal{O}(\Delta 2^n)}$

May or may not be long, depending on strength of the drive fields

Hastrup et al., Phys. Rev. Lett. 128, 110503 (2022).

YL et al., arXiv:2408.14729.

Numerical Simulation of State Transfer Protocols

Single-variable QSP

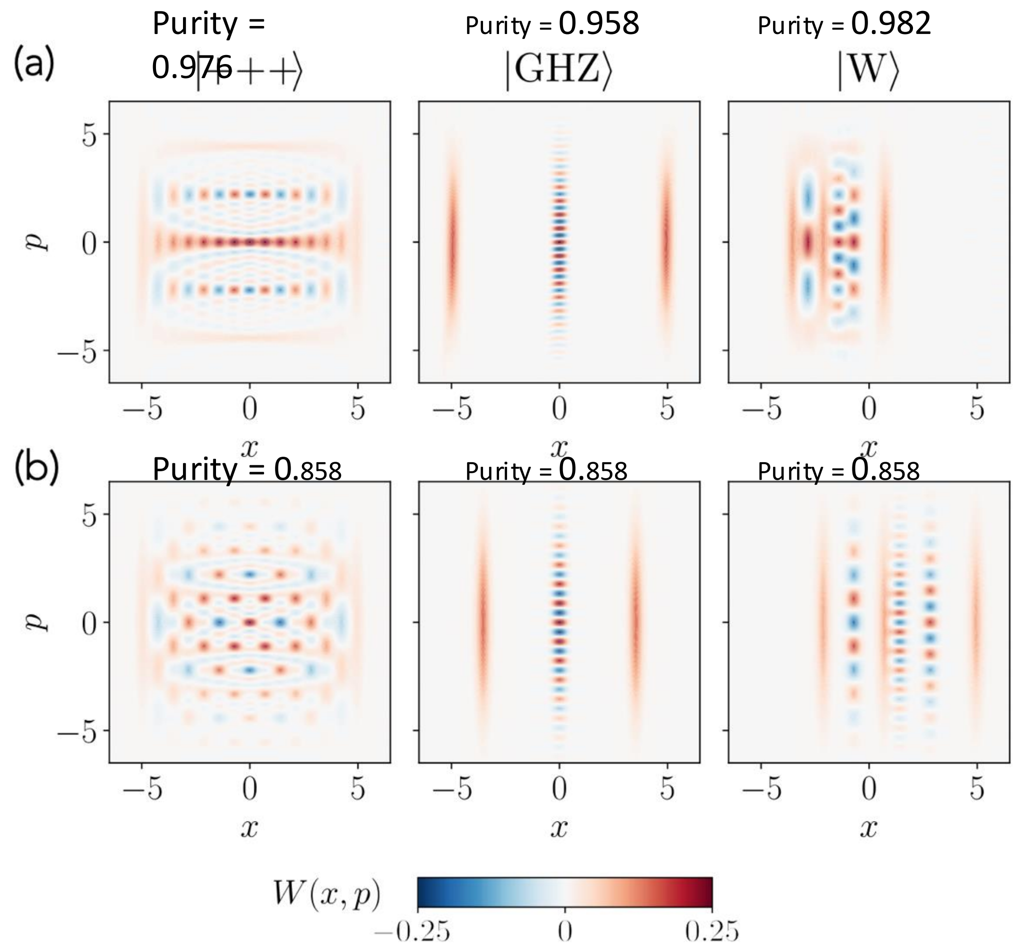
d=60 with δ=0.2, Δ=1
$\sigma = e^{-1.12} \approx 0.37$

$\boldsymbol{d}$: polynomial degree. $\boldsymbol{\delta}$: rising edge width of QSP ploy.
$\boldsymbol{\sigma}$: width of initial Gaussian Δ: spacing between Gauss.

Non-abelian QSP

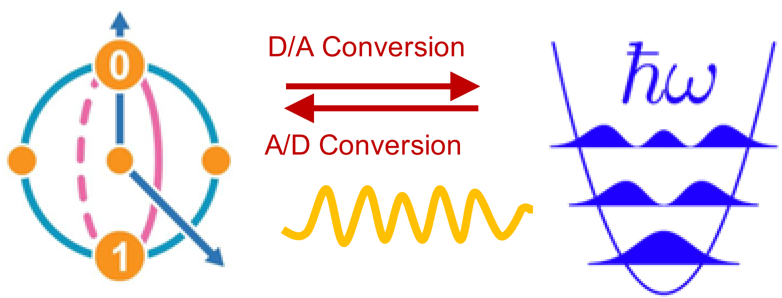
$\Delta = \sqrt{2}$, sinc is approximated as Gaussian with width $\sigma = e^{-1.12} \approx 0.37$

Fock-level truncation = 128

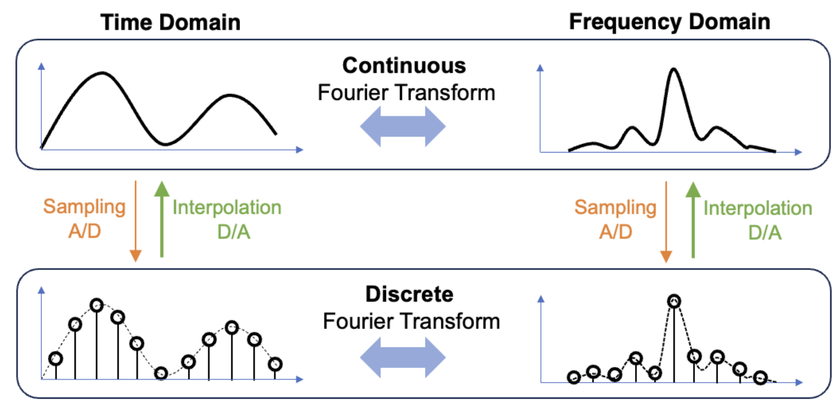


Signal Processing Perspective: State Transfer = Quantum AD/DA Conversion

Digital Quantum "Signals" **Analog** Quantum "Signals"



Classical Signal Types



Position-Momentum dual ↔ **Time-Frequency dual**

A Mixed Analog-Digital Quantum Signal Processing framework!

See arXiv:2408.14729 for more details.

Application II: Quantum Fourier Transform

- Certain tasks can be extremely easy on CV systems.

Quantum Fourier Transform (*n* qubits)

$$|\psi\rangle_Q = \sum_{\mathbf{x}} c_{\mathbf{x}} |\mathbf{x}\rangle_Q$$

$$U_{\mathrm{QFT}}|\psi\rangle_Q = \sum_{\mathbf{x}} \left[\sum_{\mathbf{y}} \frac{1}{\sqrt{2^n}} c_{\mathbf{y}} e^{2\pi i x y / 2^n} \right] |\mathbf{x}\rangle_Q$$

Gates: $O(n^2)$, exact; $O(n\,\mathrm{polylog}(n))$, approximation.

$$\boldsymbol{F}^\dagger \,\hat{\boldsymbol{x}}\, \boldsymbol{F} = e^{i\frac{\pi}{2}\hat{n}} \,\frac{\boldsymbol{a} + \boldsymbol{a}^\dagger}{\sqrt{2}}\, e^{-i\frac{\pi}{2}\hat{n}}$$

$$= \frac{e^{-i\frac{\pi}{2}}\,\boldsymbol{a} + e^{i\frac{\pi}{2}}\,\boldsymbol{a}^\dagger}{\sqrt{2}} = \frac{\boldsymbol{i}(\boldsymbol{a}^\dagger - \boldsymbol{a})}{\sqrt{2}} = \hat{\boldsymbol{p}}$$

(use BCH)

Osc. free-evolution = Fourier transform

$$F = e^{-i\frac{\pi}{2}\hat{n}}$$

$$\hat{\boldsymbol{x}} \quad \xrightarrow{\qquad} \quad \hat{\boldsymbol{p}}$$

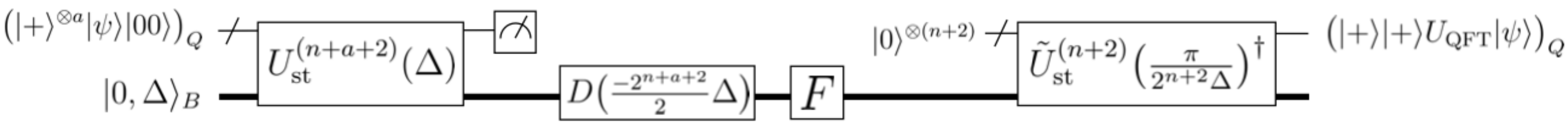
If *n*-qubit data can be encoded in the $\boldsymbol{x}$-quadrature, **only need 1 gate to perform the QFT**!

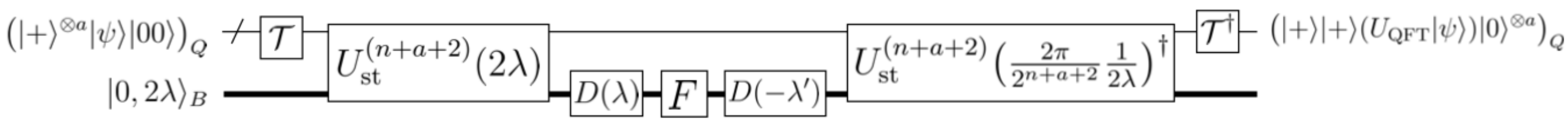
Use DV-CV State Transfer Protocols!

$\hat{n} = \hat{a}^\dagger \hat{a}$: number operator
$\hat{\boldsymbol{x}}$: position operator, $\hat{\boldsymbol{p}}$: momentum operator

Application II: QFT from Oscillator Free Evolution

- QFT with state transfer based on **single-variable**



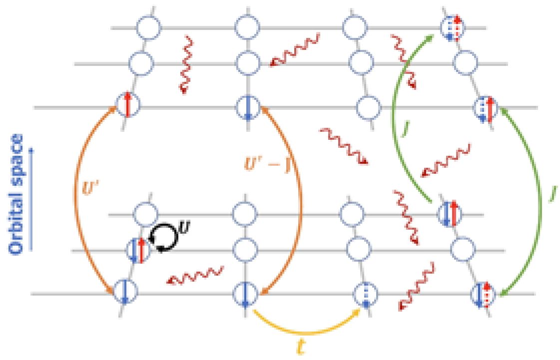


Gate count: O(n) instead of O(n^2) or O(n polylog(n)) as in qubit QFT.

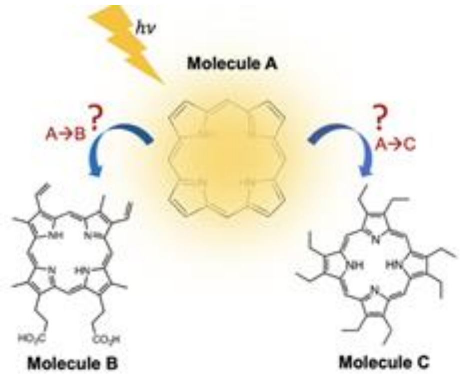
Caveat: Runtime; approximation

See arXiv:2408.14729 for more details

Application (dynamics): Dissipative Vibronic Dynamics



electron-**phonon** coupling



electron-**vibrational** coupling

Quantum simulation of boson-fermi mixtures

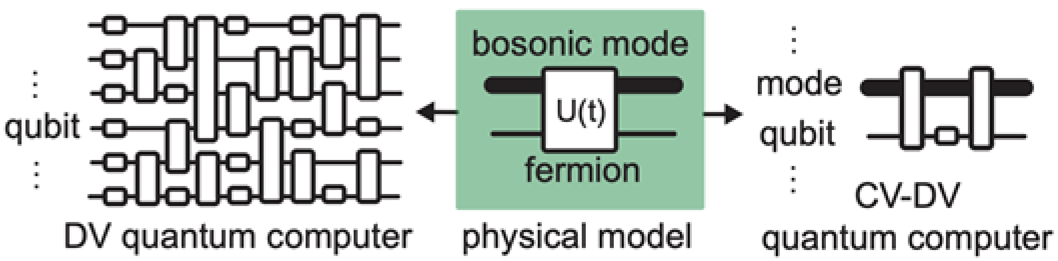
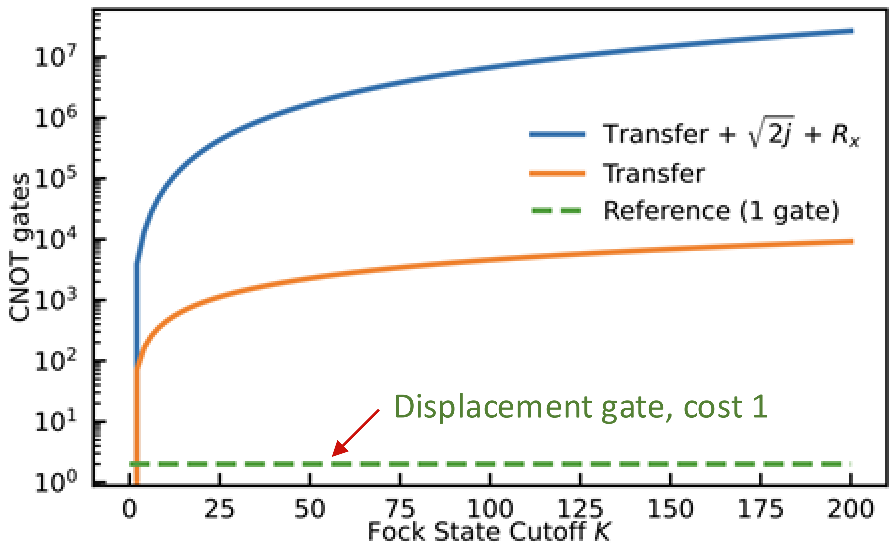


Figure from E. Crane et al., arXiv: 2409.03747

Application (dynamics): Advantage for Simulating Bosonic Matter

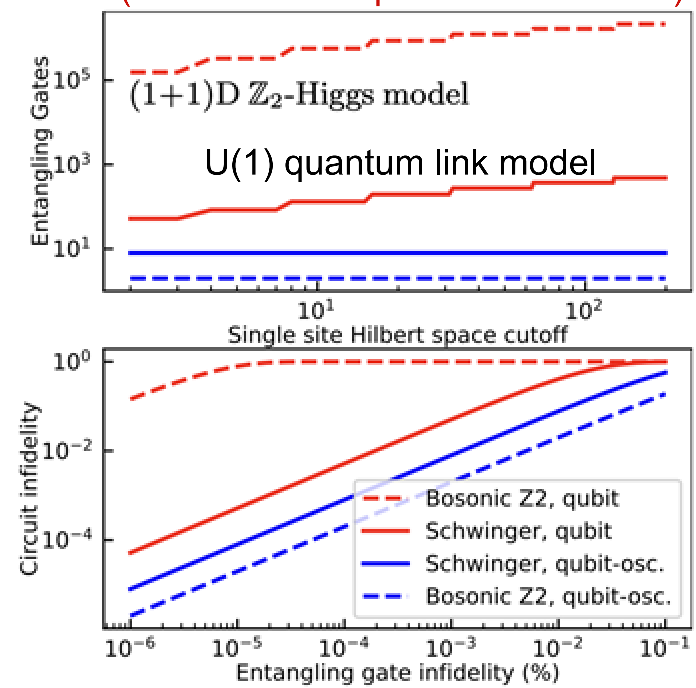
Complexity of simulating a displacement gate $D(\alpha)$ with qubit gate sets



$$\hat{x} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 1 & 0 & \sqrt{2} & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \cdots \\ 0 & 0 & \sqrt{3} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

YL, S. Singh, K.C. Smith et al. arXiv: 2407.10381.

Simulation Cost of two different fermi-boson models on qubits vs. qubit-oscillators
(One Trotter step for 2 matter sites)

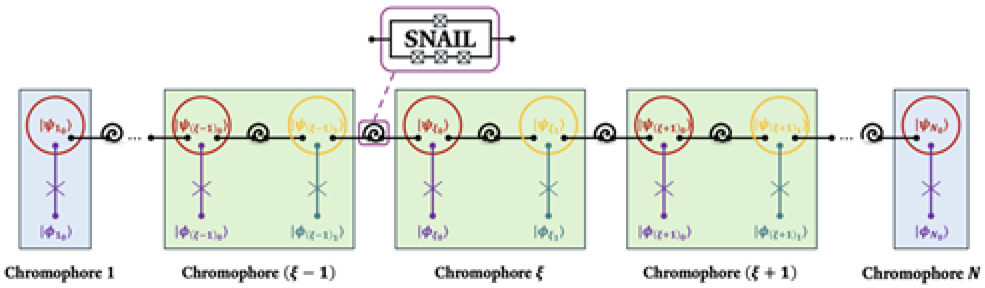


E. Crane et al. arXiv: 2409.03747

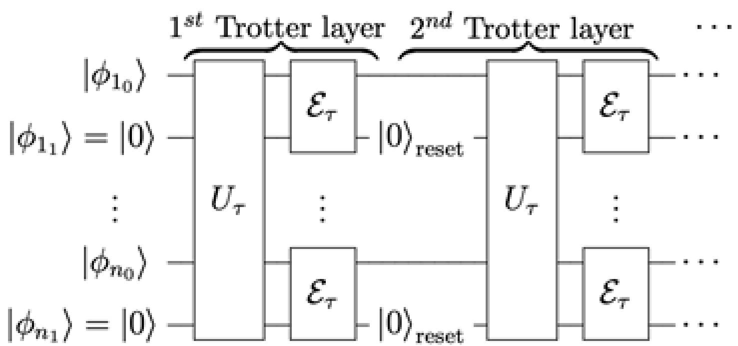
Application (dynamics): Compilation of Dissipative Non-Adiabatic Vibronic Dynamics on cQED Devices

Generalization to N-chromophores

(2N transmons + 2N cavity)



Electronic Dissipation via Reset



Gate cost of 1 Trotter step (0.01 pico-second of the real vibronic dynamics):

$$N_{\text{gate}} = (N-2) \times (336\ \text{BS} + 345\ \text{CD} + 3\ \text{SNAP})$$

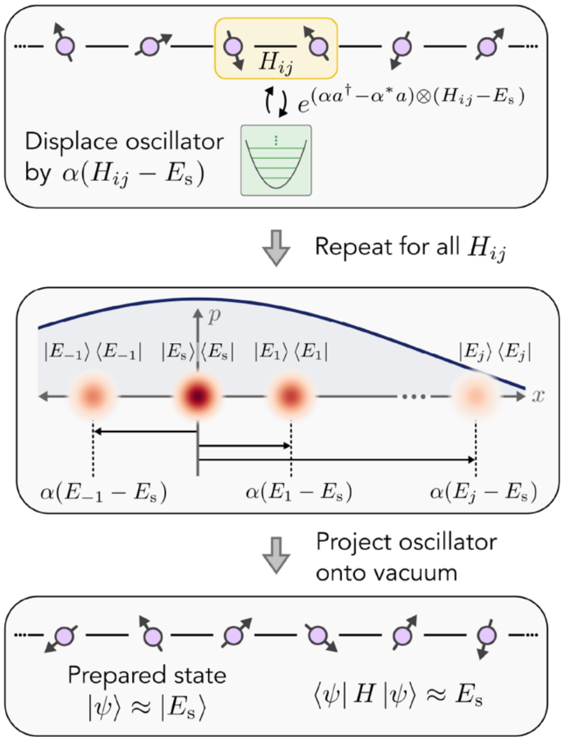
Mid-circuit measurement and reset

Effect of hardware noise~ pico-second vibronic dynamics simulation is possible, if gate fidelity > 99.99%

See arXiv:2502.17820 (accepted to JCTC) for more details. Collaboration with Victor Batista (Yale Chemistry)

Co-Designing Eigen- and Singular-value Transformation Oracles: From Algorithmic Applications to Hardware Compilation

Luke Bell et al., [arXiv:2502.16029](https://arxiv.org/abs/2502.16029)



Using qubit-controlled oscillator displacements to block encode a Gaussian energy filter

(1) Map local energy terms $\left(\hat{H}-E_s\right)$ of Heisenberg spin chain onto an ancilla qubit

(2) Ancilla controlled displacement of oscillator

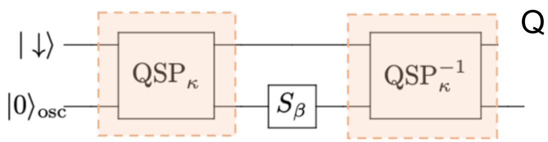
$$\left|\Psi_{\text{spins}}\right\rangle \otimes |0\rangle \rightarrow e^{i\alpha\left(\hat{H}-E_s\right)\hat{p}} \left|\Psi_{\text{spins}}\right\rangle \otimes |0\rangle$$

(3) Project displaced oscillator onto vacuum (by measurement)

$$\left|\Psi_{\text{spins}}\right\rangle \rightarrow e^{-\frac{\alpha^2}{2}\left(\hat{H}-E_0\right)^2} \left|\Psi_{\text{spins}}\right\rangle$$

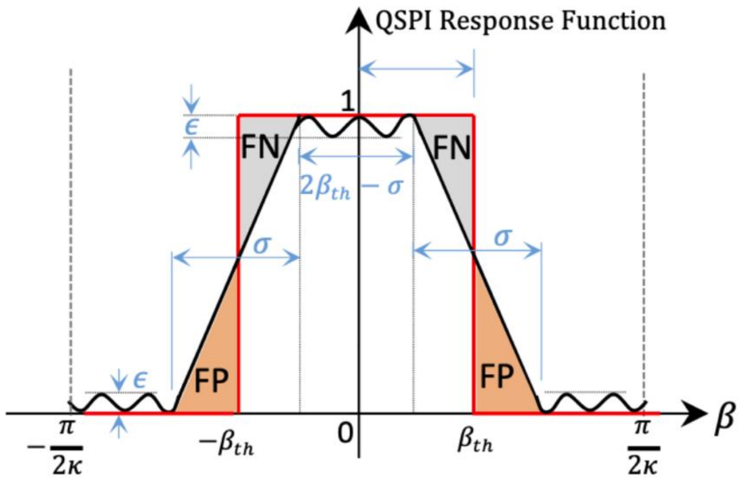
Single-Shot Quantum Detection

Nonlinear transformation of the sensing parameter in single-shot limit.



QSPI Response Function(β)

Qubit measurement probability vs. β



FN: false-negative error
FP: false-positive error

- ✓ Step function will be ideal (red)
- ✓ Actual qubit response function (black)
- ✓ Minimize error decision probability (shaded)

How to quantify decision quality:

$$p_{\mathrm{err}}(\beta_{\mathrm{th}}, k) = \frac{k}{\pi}\int_{-\frac{\pi}{2k}}^{\frac{\pi}{2k}} |P_{\mathrm{approx}}(\beta) - P_{\mathrm{ideal}}(\beta)| d\beta$$
$$= p_{\mathrm{err,FN}}(\beta_{\mathrm{th}}) + p_{\mathrm{err,FP}}(\beta_{\mathrm{th}}).$$

Efficient scaling on decision error (single shot)

$$p_{\mathrm{err}} \propto \frac{1}{kd}\log(d)$$

d is the circuit depth

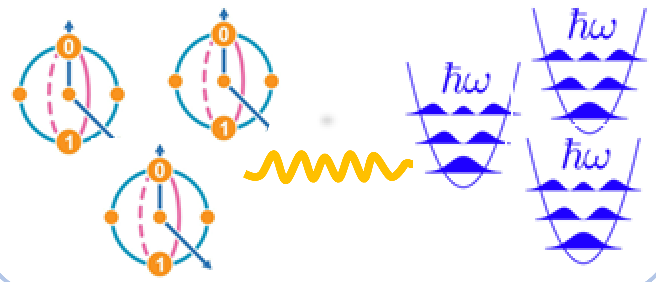
Jasmine Sinanan-Singh et al., Quantum 8, 1427 (2024).

What's next?

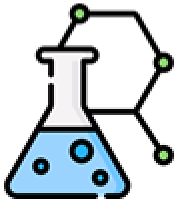
DV-CV Variational Alg.

Early fault-tolerance from QEC

Hybrid CV-DV Quantum Computation

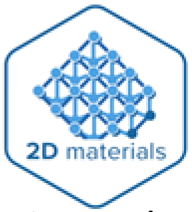


Optimization

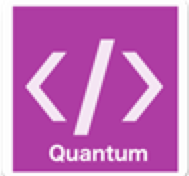


Chemistry

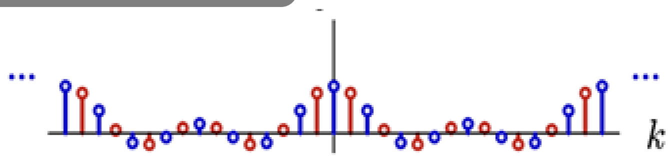
Metrology/Sensing



Materials



Programming languages



Digital-analog quantum signal processing

Summary and Open Questions

- DV (qubits) + CV (oscillators) => a powerful paradigm of hybrid CV-DV quantum computing
- Physical ISAs and AMMs have been established => New applications

- **Open questions:**
  - Control: Complete theory of non-Abelian QSP; Error propagation
  - Algorithms: more algorithmic framework, measurement-feedforward
  - Applications: more general quantum arithmetic
  - Fault-tolerance and architecture: Logical Hybrid CV-DV ISAs; parallelism; quantum memory; benchmark & certification
  - Advantage: Compare with qubit algorithms

C^{2}QA ISA (theory) collaboration

Nathan Wiebe
U. Toronto & PNNL

Tim Stavenger
PNNL

Chris Kang
U. Washington -> Chicago

Eleanor Crane
UCL -> Maryland, MIT

Micheline Solely
Yale -> Wisconsin

Kevin Smith
Yale -> IBM

+ Chuang group (MIT) John Martyn
+ Yuan Liu (MIT->NC State)

+ Alec Eickbusch (Yale -> Google)

+Shraddha Singh (Yale)
+Baptiste Royer (Yale->Sherbrooke)

ORNL/Yale/NC State Gaussian Energy Filter collaboration:

Luke Bell, Yan Wang, Kevin C. Smith, Yuan Liu, Eugene Dumitrescu, SMG

arXiv:2502.16029

arXiv:2407.10381

arXiv:2209.11153

- Instruction Set Architecture for hybrid qubit/oscillator systems
- "Bosonic Qiskit" extension to oscillators
  - Represent $\Lambda = 2^n$ levels of oscillator with a register of $n = \log_2 \Lambda$ qubits
  - Access ISA and Wigner tomography toolkit within Qiskit

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MIT Physics & EECS

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MIT Physics

Jasmine Sinanan-Singh
MIT Physics

Gabriel Mintzer
MIT Physics

Kevin Smith
Yale -> IBM

Victor Batista
Yale Chemistry

Nam Vu
Yale Chemistry

Xiaohan Dan
Yale Chemistry

Daniel Dong
NC State CSC

NC STATE UNIVERSITY
NTTResearch
U.S. DEPARTMENT OF ENERGY
C2QA
Massachusetts Institute of Technology

NC STATE UNIVERSITY Electrical and Computer Engineering

NC STATE UNIVERSITY COMPUTER SCIENCE

- Quantum Engineering and Simulation Theory (QUEST)
  - quantum algorithms and simulation
  - hybrid continuous-discrete-variable quantum computing
  - quantum engineering

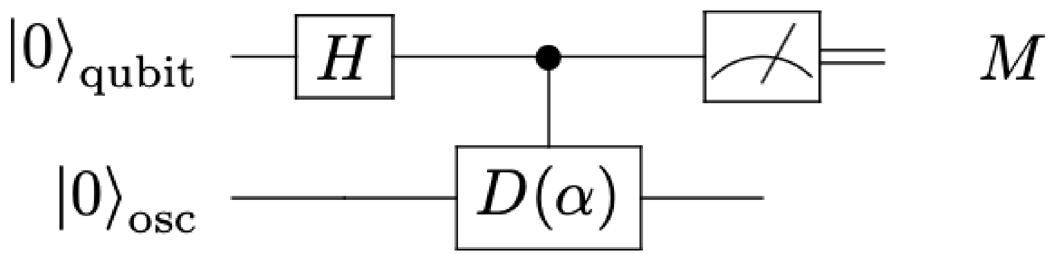
- Email: yliu335@ncsu.edu
- https://yuanliu.group

Thanks!

References

- ‘Hybrid Oscillator-Qubit Quantum Processors: Instruction Set Architectures, Abstract Machine Models, and Applications,’ Yuan Liu, Shraddha Singh, Kevin C. Smith et al., arXiv:2407.10381. Under review on PRX Quantum.
- ‘Hybrid Oscillator-Qubit Quantum Processors: Simulating Fermions, Bosons, and Gauge Fields,’ Eleanor Crane et al., arXiv:2409.03747
- ‘Toward Mixed Analog-Digital Quantum Signal Processing: Quantum AD/DA Conversion and the Fourier Transform,’ Yuan Liu et al., arXiv:2408.14729. Under review on IEEE TSP.
- ‘Unification of Finite Symmetries in Simulation of Many-body Systems on Quantum Computers’, Victor M Bastidas, Nathan Fitzpatrick, KJ Joven, Zane M Rossi, Shariful Islam, Troy Van Voorhis, Isaac L Chuang, Yuan Liu. Phys. Rev. A 111, 052433 (2025).
- ‘Co-Designing Eigen- and Singular-value Transformation Oracles: From Algorithmic Applications to Hardware Compilation,’ Luke Bell et al., arXiv:2502.16029. Under review on PRX Quantum.
- ‘Bosonic Qiskit,’ Timothy J Stavenger, Eleanor Crane, Kevin Smith et al., arXiv:2209.11153

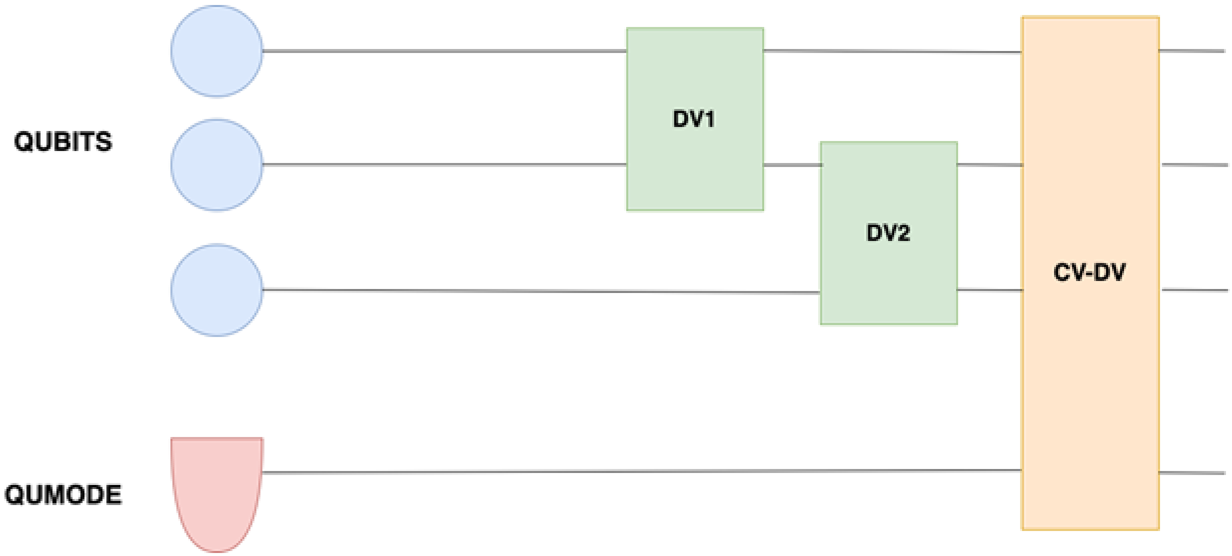
Illustration of Programming Hybrid CV-DV Circuits




```
% Initialize qumode register
qmr = c2qa.QumodeRegister(num_qumodes = 1,
            num_qubits_per_qumode = 6)
% Initialize qubit register
qbr = qiskit.QuantumRegister(1)
% Initialize classical register
cr = qiskit.ClassicalRegister(1)
% Initialize hybrid CV-DV quantum circuit
circuit = c2qa.CVCircuit(qmr, qbr, cr)
```

```
% Assign gate parameters for CV gates
alpha = 1
% Append a DV gate
circuit.h(qbr[0])
% Append a hybrid CV-DV gate
circuit.cv_c_d(alpha, qmr[0], qbr[0])
% Measure CV and DV components
circuit.measure(qbr[0], cr[0])
```

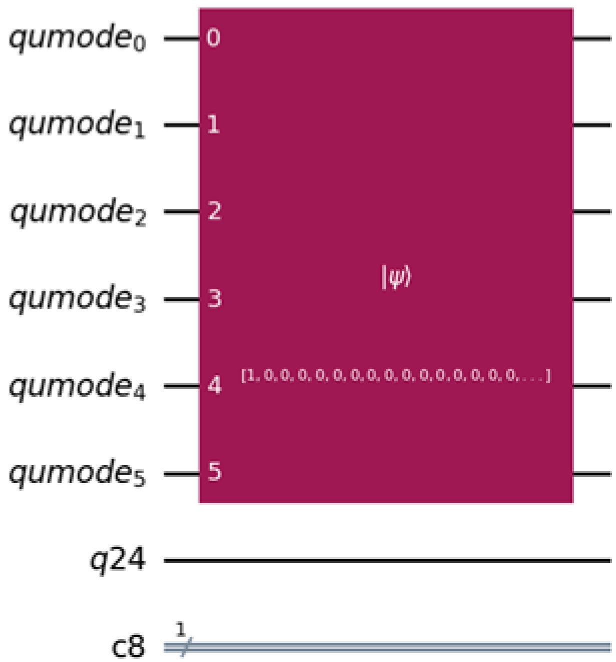
Coding and Programming Demontration for Hybrid CV-DV Quantum Processors



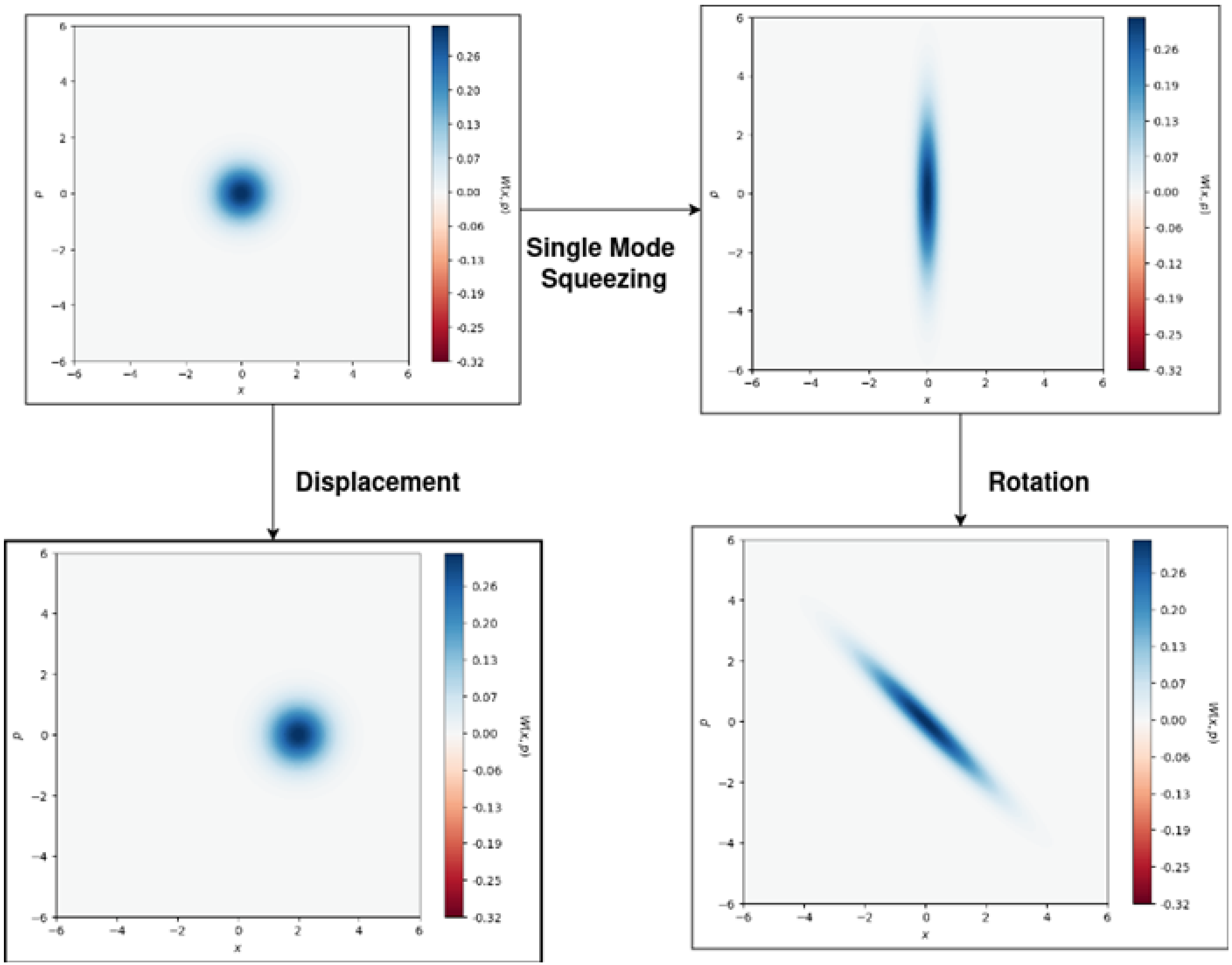
https://github.com/shubdeepmohapatra01/ISCA2025-Tutorial/tree/main

Bosonic Qiskit

- Simulation Framework for CV-DV systems.
- Each qumode is made up of multiple qubits
- For a qumode with n qubits the Fock level cutoff is 2^n



Basic gates



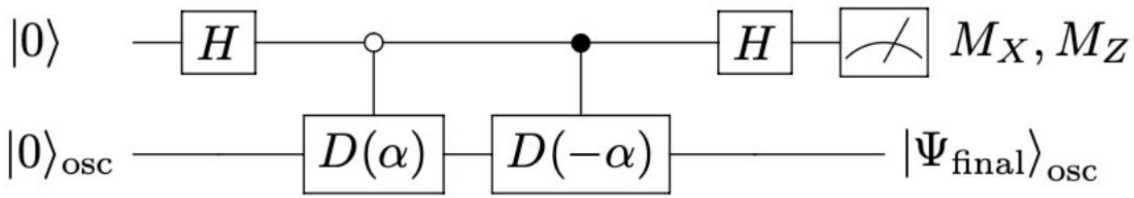
CAT State

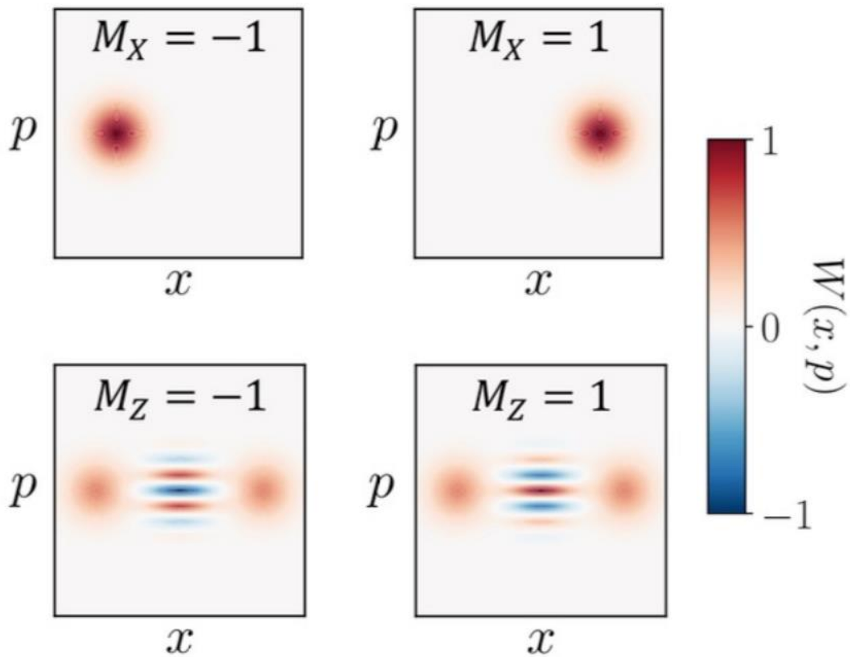
- CAT state is a quantum superposition of two coherent states

  Cat = N ($\alpha \pm -\alpha$)
- Even cat state: + sign → constructive interference
- Odd cat state: − sign → destructive interference

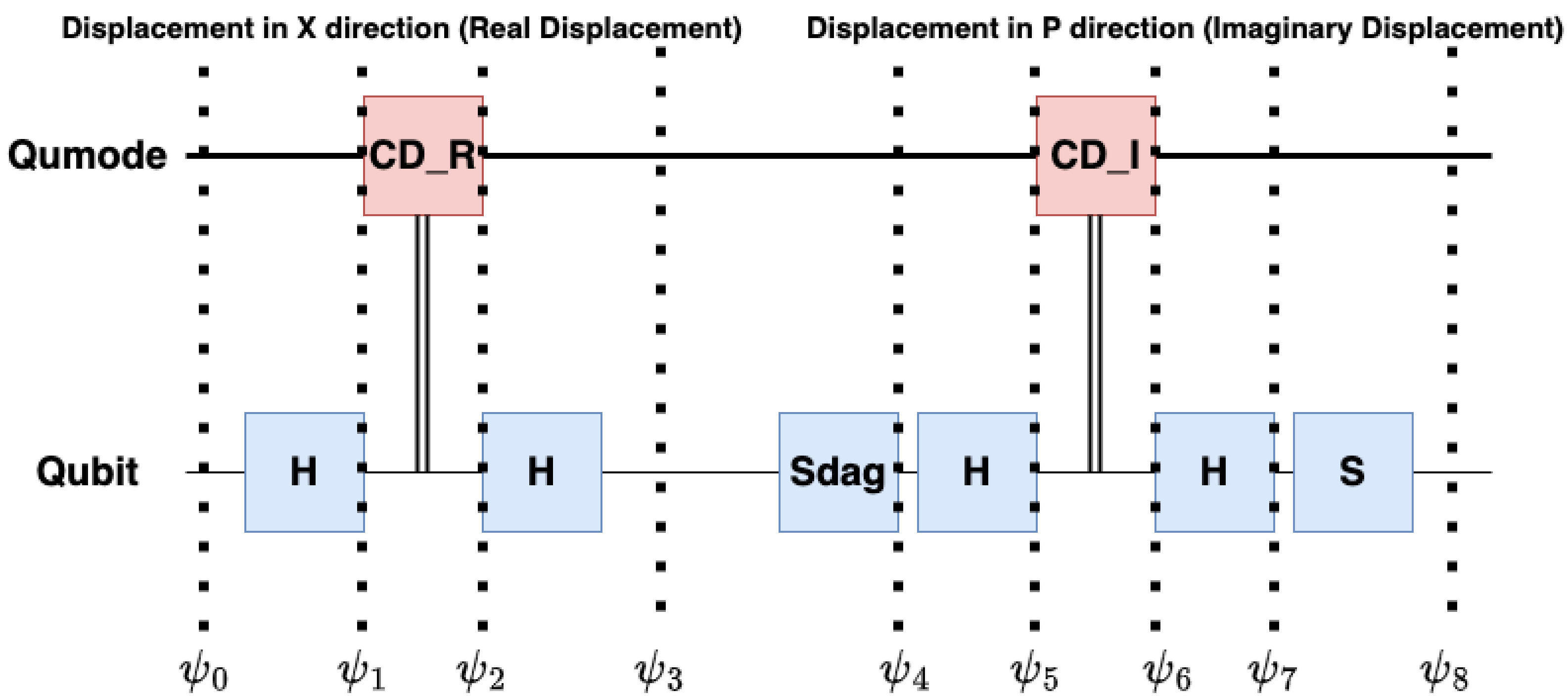
- Why is it useful?
  - Encoding logical qubits into cat states (cat qubits)
  - Generating GKP codewords

Nondeterministic CAT State Generation





CAT State


Displacement in X direction (Real Displacement)
Displacement in P direction (Imaginary Displacement)
Qumode
Qubit
CD_R
CD_I
H
H
Sdag
H
H
S
ψ_0
ψ_1
ψ_2
ψ_3
ψ_4
ψ_5
ψ_6
ψ_7
ψ_8

CAT State

- $$\Psi_0 = |0\rangle_{osc}|0\rangle$$
- $$\Psi_1 = |0\rangle_{osc}|+\rangle$$
- Apply conditional displacement in the x direction (real displacement) α

$$\Psi_2 = \frac{1}{\sqrt{2}}\left(|\alpha\rangle_{osc}|0\rangle + |-\alpha\rangle_{osc}|1\rangle\right)$$

- $$\Psi_3 = \frac{1}{2}\left(\left(|\alpha\rangle_{osc}| + |-\alpha\rangle_{osc}\right)|0\rangle + \left(|\alpha\rangle_{osc}| - |-\alpha\rangle_{osc}\right)|1\rangle\right)$$
- Let $(|\alpha\rangle_{osc}| + |-\alpha\rangle_{osc}) = C_+$ and $(|\alpha\rangle_{osc}| - |-\alpha\rangle_{osc}) = C_-$

$$\Psi_4 = \frac{1}{2}\left(C_+|0\rangle - iC_-|1\rangle\right)$$

CAT State

- $$\Psi_5 = \frac{1}{2\sqrt{2}}\left((C_+ - iC_i)\,|0\rangle + (C_+ + iC_i)\,|1\rangle\right)$$

- Apply conditional displacement in the p direction (imaginary displacement) $i\frac{\pi}{8\alpha}$

$$\Psi_6 = \frac{1}{2\sqrt{2}}\left(\left(a|\alpha + i\frac{\pi}{8\alpha}\rangle osc| + b| - \alpha + i\frac{\pi}{8\alpha}\rangle osc\right)|0\rangle + \left(b|\alpha - i\frac{\pi}{8\alpha}\rangle osc| + a| - \alpha - i\frac{\pi}{8\alpha}\rangle osc\right)|1\rangle\right)$$

where a = 1-i and b = 1+i

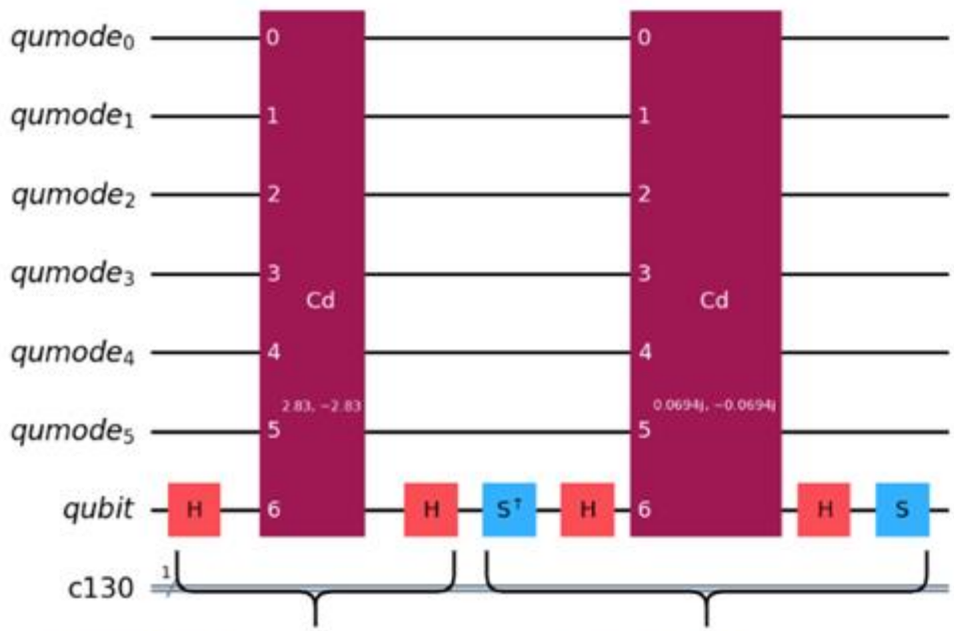
- $$\Psi_7 = \frac{1}{4}\left(\left(a|\alpha + i\frac{\pi}{8\alpha}\rangle osc| + b|\alpha - i\frac{\pi}{8\alpha}\rangle osc + b| - \alpha + i\frac{\pi}{8\alpha}\rangle osc| + a| - \alpha - i\frac{\pi}{8\alpha}\rangle osc\right)|0\rangle + \left(a|\alpha + i\frac{\pi}{8\alpha}\rangle osc| - b|\alpha - i\frac{\pi}{8\alpha}\rangle osc + b| - \alpha + i\frac{\pi}{8\alpha}\rangle osc| - a| - \alpha - i\frac{\pi}{8\alpha}\rangle osc\right)|1\rangle\right)$$

- $$\Psi_8 = \frac{1}{4}\left(\left(a|\alpha + i\frac{\pi}{8\alpha}\rangle osc| + b|\alpha - i\frac{\pi}{8\alpha}\rangle osc + b| - \alpha + i\frac{\pi}{8\alpha}\rangle osc| + a| - \alpha - i\frac{\pi}{8\alpha}\rangle osc\right)|0\rangle + \left(b|\alpha + i\frac{\pi}{8\alpha}\rangle osc| + a|\alpha - i\frac{\pi}{8\alpha}\rangle osc - a| - \alpha + i\frac{\pi}{8\alpha}\rangle osc| - b| - \alpha - i\frac{\pi}{8\alpha}\rangle osc\right)|1\rangle\right)$$

NC STATE UNIVERSITY

CAT State

```python
Nqubits = 1
cutoff = 2**6
alpha = 4

qmr = c2qa.QumodeRegister(num_qumodes=1, num_qubits_per_qumode=int(np.ceil(np.log2(cutoff))),name='qumode')
qbr = QuantumRegister(Nqubits,name='qubit')
cbr = ClassicalRegister(Nqubits)
circuit = c2qa.CVCircuit(qmr, qbr,cbr)
```




```python
circuit.h(qbr[0])
circuit.cv_c_d(alpha / np.sqrt(2),qmr[0],qbr[0])
circuit.h(qbr[0])
```

```python
circuit.sdg(qbr[0])
circuit.h(qbr[0])
circuit.cv_c_d(1j*np.pi/(8*alpha*np.sqrt(2)),qmr[0],qbr[0])
circuit.h(qbr[0])
circuit.s(qbr[0])
```

GKP State

- GKP States:

  $|0\rangle_{(GKP)} \propto \sum_{m} |\, m\sqrt{\pi}\, \rangle_{x}$

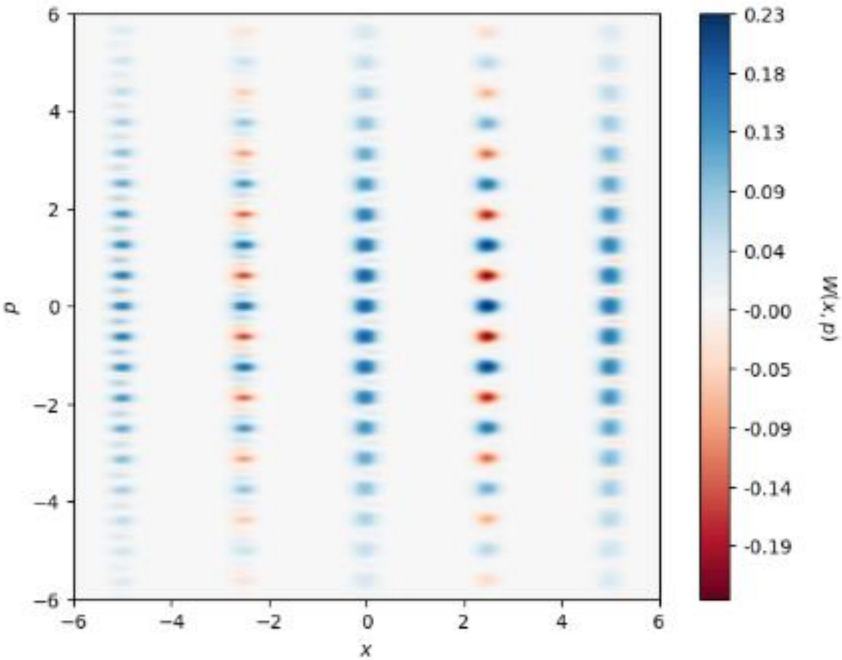
  $|1\rangle_{(GKP)} \propto \sum_{m} |\, (m+1)\sqrt{\pi}\, \rangle_{x}$

- Multiple rounds of cat-state protocol with displacement $\alpha = \sqrt{\pi}$
- After k rounds, each controlled displacement adds 2 new peaks.
- The result is a superposition of 2^k equally spaced peaks in position space.

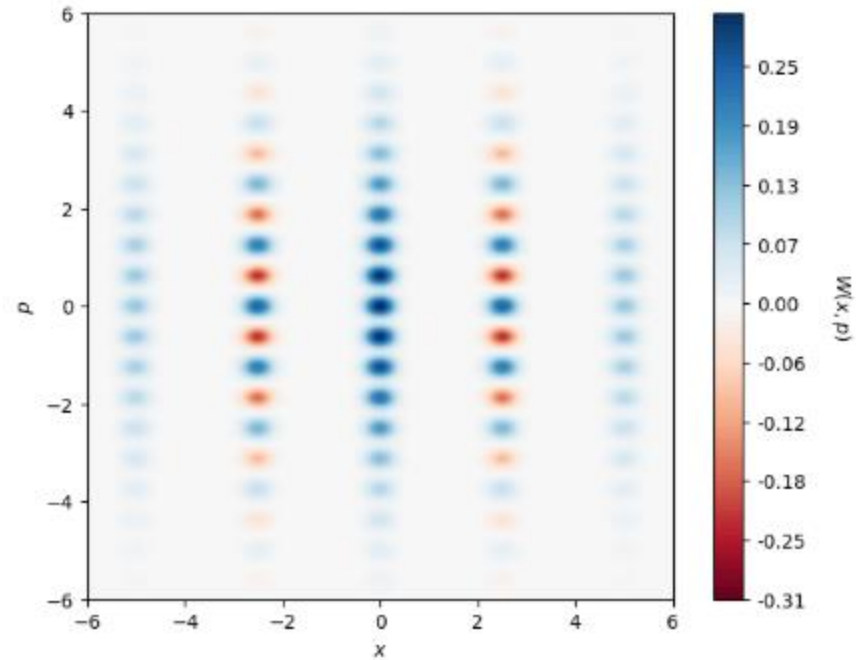
Why GKP State is useful?

- Error Correction : Can correct errors in both position and momentum
- Fault Tolerant Logical Qubit Encoding
- Used as input state for CV-DV applications like Shor's algorithm

GKP State

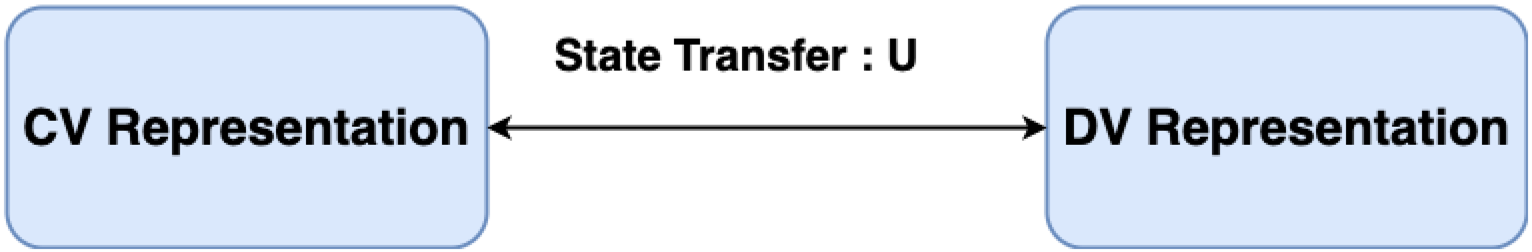


Approximate GKP State



Ideal GKP State

State-Transfer

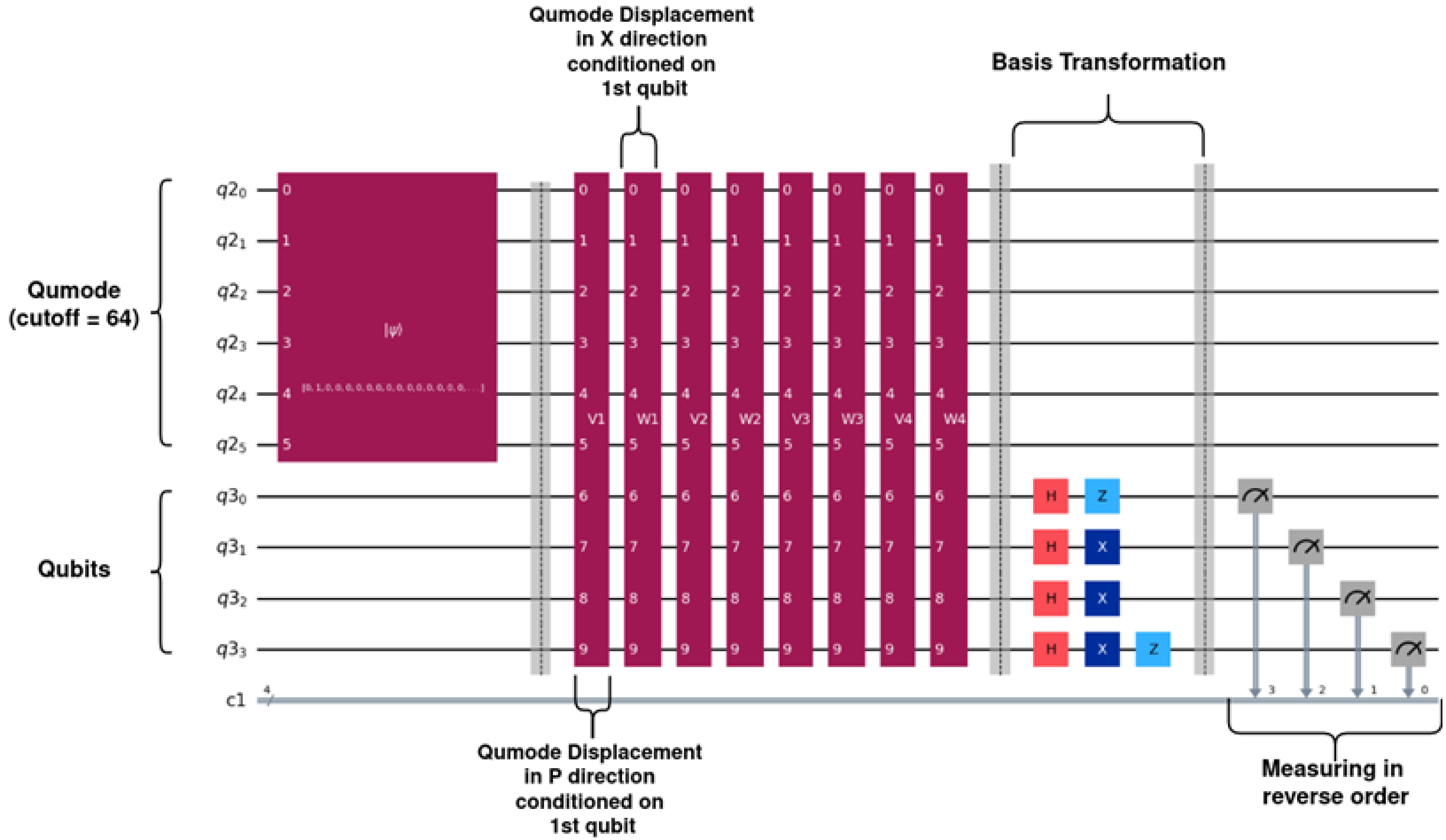


$$U_{\text{D/A}}^{\text{N-A}}(\Delta, n)^{\dagger} = \prod_{j=1}^{n} V_j W_j = V_n W_n \cdots V_1 W_1,$$

$$V_j = e^{i \frac{\pi}{2^j \Delta} \hat{x} \hat{\sigma}_y^{(j)}}, \quad W_j = \begin{cases} e^{i \frac{\Delta}{2} 2^{j-1} \hat{p} \hat{\sigma}_x^{(j)}} & j < n, \\ e^{-i \frac{\Delta}{2} 2^{j-1} \hat{p} \hat{\sigma}_x^{(j)}} & j = n, \end{cases}$$

[1] Liu, Yuan, et al. "Toward Mixed Analog-Digital Quantum Signal Processing: Quantum AD/DA Conversion and the Fourier Transform." *arXiv preprint arXiv:2408.14729* (2024).

CV to DV State Transfer Circuit



CV to DV State Transfer

```python
qmr = c2qa.QumodeRegister(1, num_qubits_per_qumode=6)
qbr1 = QuantumRegister(4)
cr1 = ClassicalRegister(4)
circuit = c2qa.CVCircuit(qmr, qbr1, cr1)
```

```python
# Apply basis transformation
apply_basis_transformation(circuit, qbr1)
```

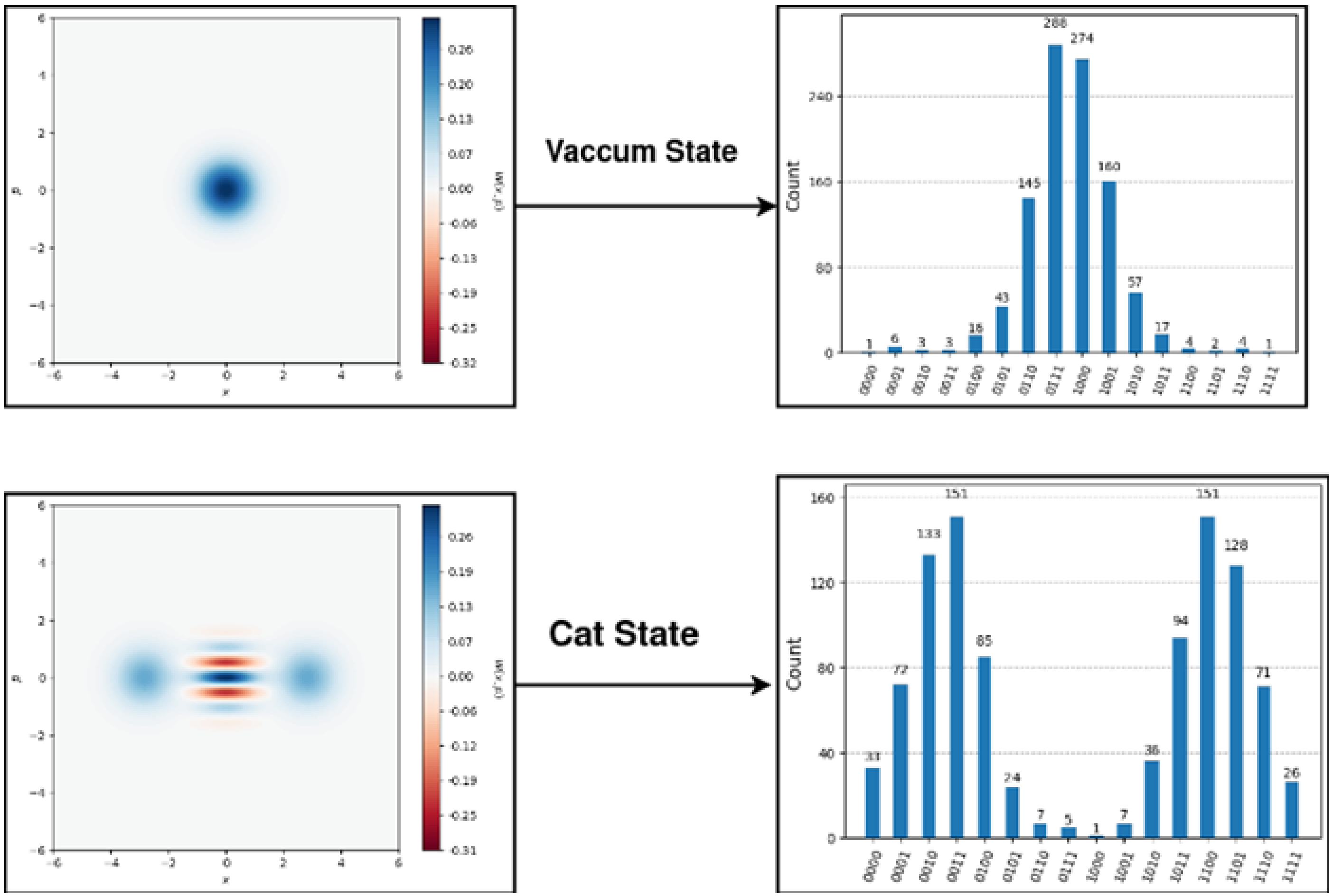
```python
def apply_basis_transformation(circuit, qbr1):
    num_qubits = len(qbr1)
    for i in range(num_qubits):
        circuit.h(qbr1[i])
        if i == num_qubits - 1:  # MSB
            circuit.x(qbr1[i])
            circuit.z(qbr1[i])
        elif i == 0:  # LSB
            circuit.z(qbr1[i])
        else:  # Middle qubits
            circuit.x(qbr1[i])
```


```python
# Simulate and measure
for i in range(len(qbr1)):
    circuit.measure(qbr1[i], cr1[-(i + 1)])

state1, result, _ = c2qa.util.simulate(circuit, shots=1024)
```

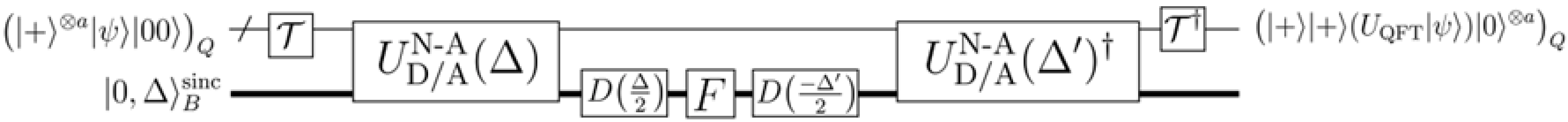
```python
# Add the CV-to-DV transfer gate
for j in range(1,qbm+1):
        V_j = state_transfer.Vj(lmbda,j,4,cutoff)
        gate1 = UnitaryGate(V_j.full(), label=f'V{j}')
        circuit.append(gate1, qmr[:] + qbr1[:])  # adding custom gate : Conditional displacement in p direction

        W_j = state_transfer.Wj(lmbda,j,4,cutoff)
        gate1 = UnitaryGate(W_j.full(), label=f'W{j}')
        circuit.append(gate1, qmr[:] + qbr1[:])  # adding custom gate : Conditional displacement in x direction
```

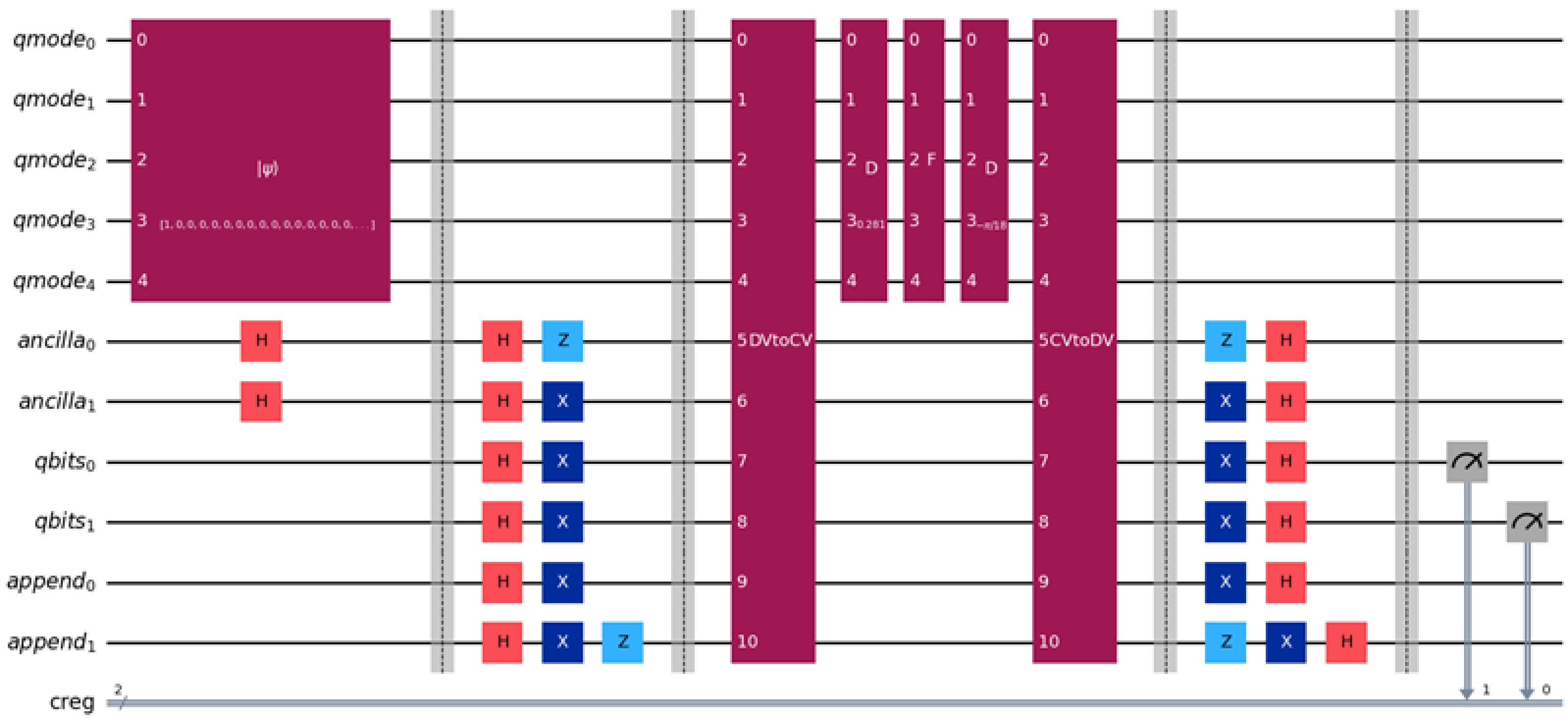
```python
def Vj(lmbda,j,n,cutoff):
    qubit_pauli = tensor([identity(2)]*(j-1) + [sigmay()] + [identity(2)]*(n-j))
    return tensor(qubit_pauli, (1j*np.pi)/(2**(j+1)*lmbda)*(destroy(cutoff) + create(cutoff))/np.sqrt(2.0)).expm()

def Wj(lmbda,j,n,cutoff):
    qubit_pauli = tensor([identity(2)]*(j-1) + [sigmax()] + [identity(2)]*(n-j))
    if j == n:
        disp_amount = -lmbda*2**(j-1)
    else:
        disp_amount = lmbda*2**(j-1)
    return tensor(qubit_pauli, disp_amount*(destroy(cutoff) - create(cutoff))/np.sqrt(2.0)).expm()
```

State Transfer Circuit



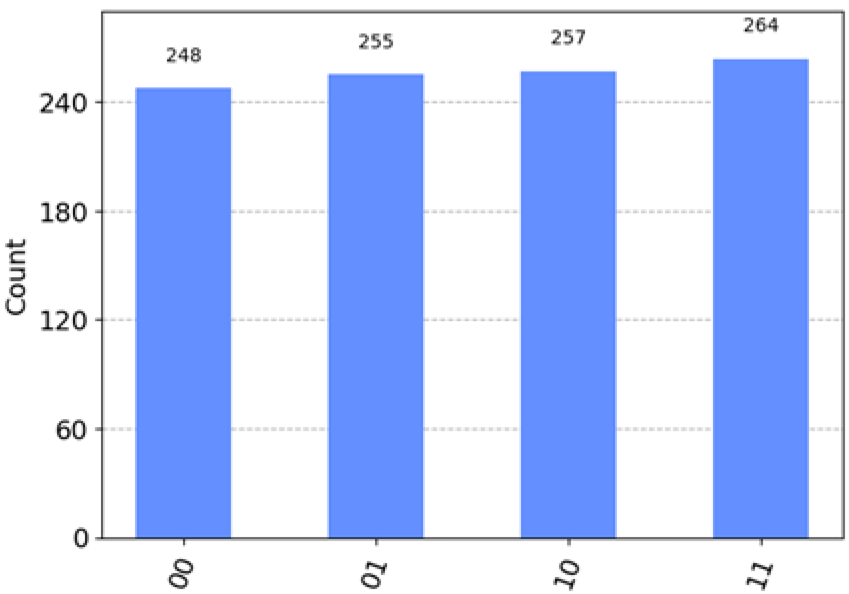
QFT



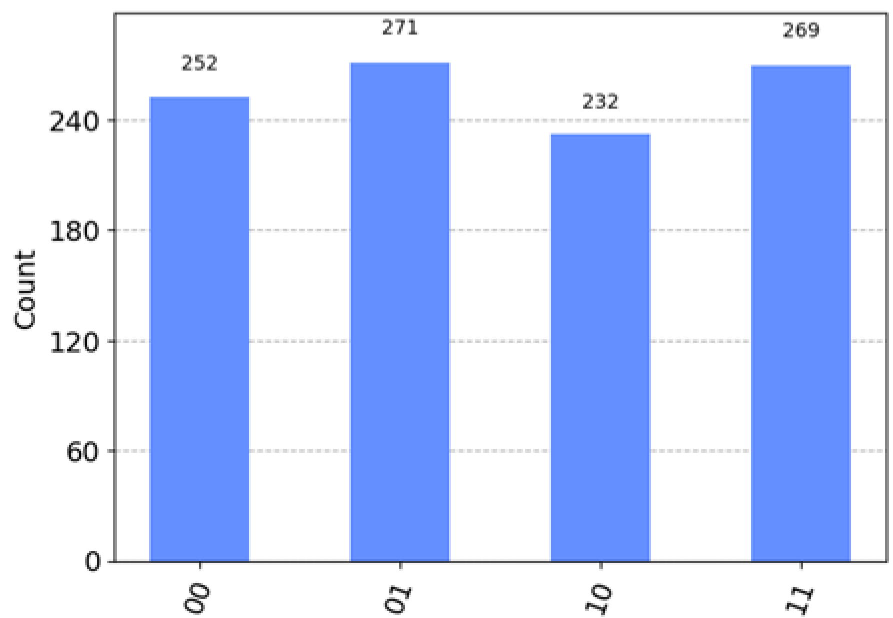


[1] Liu, Yuan, et al. "Toward Mixed Analog-Digital Quantum Signal Processing: Quantum AD/DA Conversion and the Fourier Transform." *arXiv preprint arXiv:2408.14729* (2024).

QFT DV vs CV-DV



DV QFT (Initial State: 00)



CV-DV QFT (Initial State: 00)

THANK YOU

Extra Slides Beyond Here

Hierarchy of qubit state complexities

Category	Generators	Complexity
Product states in Z basis		classical
States generated from product states via Clifford gates	Clifford generators (H,S=Sqrt(Z),CNOT)	Quantum entangled but no 'magic.' Clifford operations easy to simulate classically via stabilizer evolution
States generated from product states via Non-Clifford Gates	T=Sqrt(S) gate	'Magic' prevents efficient classical simulation but allows violation of Bell inequalities and universal computation

Hierarchy of oscillator state complexities*

*Subtlety:
Complexity of CV states depends on what measurement resources are available.

Category	Generators	Complexity
Vacuum state		Easy to make if $h\omega \; ? \; k_{B}T$
Gaussian states generated from vacuum via Gaussian gates, or incoherent mixtures of same	H(x,p) quadratic form	Easy to simulate classically via evolution of mean and covariance matrix. Easy to sample from.
Non-Gaussian (e.g. superpositions of Gaussians, not mixtures of Gaussians)	H(x,p) higher than quadratic polynomial, e.g., x^3	Wigner function negativity prevents efficient classical simulation but allows violation of Bell-like inequalities and universal computation

In order to fully control a harmonic oscillator,
we require an anharmonic object (e.g., a qubit) as an auxiliary controller.

Microwave resonator

Transmon qubit

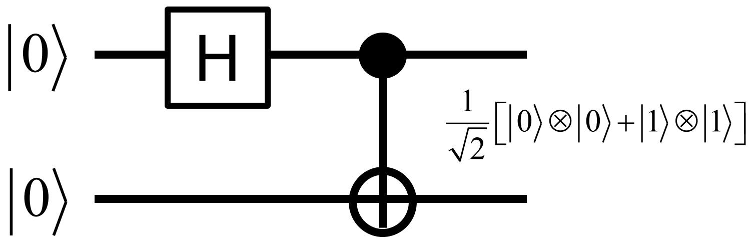
Dipole Coupling

$$H = \omega_{\mathrm{r}} a^{\dagger} a + \frac{\omega_{\mathrm{q}}}{2}\sigma^{z} + g\sigma^{x}[a + a^{\dagger}] + H_{\mathrm{damping}} \qquad \text{[Rabi]}$$

$$H = \omega_{\mathrm{r}} a^{\dagger} a + \frac{\omega_{\mathrm{q}}}{2}\sigma^{z} + g[a\sigma^{+} + a^{\dagger}\sigma^{-}] + H_{\mathrm{damping}} \qquad \text{[Jaynes-Cummings]}$$

$$\boxed{H = \omega_{\mathrm{r}} a^{\dagger} a + \frac{\omega_{\mathrm{q}}}{2}\sigma^{z} + \chi\sigma^{z} a^{\dagger} a + H_{\mathrm{damping}} \qquad \text{[Dispersive]}}$$

<u>Strong</u> Dispersive Limit



$$D(\alpha) = e^{\alpha a^\dagger - \alpha^* a} = e^{2i[\alpha_I \hat{x} - \alpha_R \hat{p}]}$$

$$U_2 D(\alpha) U_2^\dagger = e^{2i[\alpha_I \hat{x}' - \alpha_R \hat{p}']} = D(\alpha')$$